Momentum and Special Relativity

- As shown already, velocity is a useful kinematic property
- A property that is more useful (as we will see) is the linear momentum defined as

$$\vec{p} \approx m\vec{v}$$

- The momentum vector clearly points in the same direction as the velocity vector
- The units for momentum are kg m/s. There is no derived unit
- However, this relation is approximate and only valid when

$$|\vec{v}| << c$$

- Instead, we can define the exact relativistic momentum relation

$$\vec{p} = \gamma m \vec{v}$$

- where the Lorentz boost factor is given by

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{|\vec{v}|}{c}\right)^2}}$$

- which is only valid for particles with mass when $|\vec{v}| < c$

- Limits $|\vec{v}| \to c$ $\quad |\vec{v}| \to 0$

$$\gamma \to \infty \qquad \gamma \to 1$$

$$|\vec{p}| \to \infty \qquad \vec{p} \to m\vec{v}$$

classical limit

Example

A car of mass 750 kg is traveling east at a speed of 10.0 m/s. The car hits a wall and rebounds (moving west) with a speed of 0.100 m/s. Determine its momentum before and after the impact. Determine the impulse (later).

Solution:

Given: m = 750 kg,

$$\vec{\mathbf{v}}_i = \mathbf{10.0}\,\tfrac{\mathbf{m}}{\mathbf{s}}\,\hat{i}$$

$$\vec{\mathbf{v}}_f = 0.10\,\tfrac{\mathbf{m}}{\mathbf{s}}\ \mathbf{west} = \mathbf{-0.10}\,\tfrac{\mathbf{m}}{\mathbf{s}}\,\hat{i}$$

$$\vec{p}_i = m\vec{\mathrm{v}}_i = (750 \text{ kg})(10.0 \text{ m/s } \hat{i})$$
$$= 7.50\text{x}10^3 \text{ kg m/s } \hat{i}$$

$$\vec{p}_f = m\vec{\mathrm{v}}_f = (750 \text{ kg})(-0.100 \text{ m/s } \hat{i})$$
$$= -7.50\text{x}10^1 \text{ kg m/s } \hat{i}$$

- Change in momentum vector is

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i$$
$$= (-75\hat{i}) - 7500\hat{i} = -7575\hat{i} \text{ kg m/s}$$