PHYS 1311: In Class Problems
Chapter 4, Set I, Solutions
Feb. 14, 2017

Problem 1. A 1.00 kg box of Valentine chocolates sits on a table. The coefficient of static friction μ_s between the table and box is 0.2, and $\mu_k = 0.15$. (a) What is the force required to start the box moving? (b) What is the force required to keep it moving at a constant speed? (c) What is the force required to maintain an acceleration of 2.00 m/s^2? (Make sure you draw a free-body diagram).

Solution: (a) Applying Newton's 2nd Law in the y-direction gives the normal force $F_N = mg$. Apply Newton's 2nd Law in the x-direction to give

$$\Sigma F_x = F_A - f_s = ma_x = m(0) \quad (1)$$

when the box is at rest. Solving for the applied force results in

$$F_A = f_s. \quad (2)$$

When the applied force has reached a magnitude such that the box of chocolates just starts the move, the the static friction becomes equal to the maximum static friction force $f_s^{\max}$, or

$$F_A = f_s^{\max} = \mu_s F_N = \mu_s mg = (0.2)(1kg)(9.8m/s^2) \quad (3)$$

resulting in 1.96 N. (b) The equations are exactly the same once the box is moving at a constant velocity, except the maximum static friction force is replaced by the kinetic friction force to give

$$F_A = f_k = \mu_k F_N = \mu_k mg = (0.15)(1kg)(9.8m/s^2) \quad (4)$$

resulting in 1.47 N. (c) Now to keep the box accelerating in the x-direction, we return to Eq. (1) or

$$F_A - f_k = ma_x \quad (5)$$

to give

$$F_A = f_k + ma_x = \mu_k mg + ma_x = m(\mu_k g + a_x) \quad (6)$$

or

$$F_A = (1kg)(0.15 * 9.8 + 2) = 3.47 \text{ N}. \quad (7)$$

Problem 2. In the approximation that the Earth is a sphere of uniform density, it can be shown that the gravitational force it exerts on a mass m inside the Earth at a distance r from the center is $mg(r/R)$, where R is the radius of the Earth. Suppose that there is hole drilled along the diameter straight through the Earth and the air was pumped out of the hole. If UPS dropped a package of mass m on one side of the hole, how long would it take for the package to reach the other side of the Earth? Include a numerical answer. What kind of motion is this? What would happen if the person on the other side of the Earth failed to catch the package when it got to the surface?

Solution. Now, we have a radial force inside the Earth which points to the center of the Earth given by

$$\vec{F} = -\left(\frac{mg}{R} r\right)\hat{r}. \tag{8}$$

This force is a restoring force, analogous to the spring force (i.e., if we drop the box in the hole, it will travel to the other side of the earth and back displaying simple harmonic motion). So, relating this gravitational force to the spring force, we find an effective spring constant

$$k = mg/R. \tag{9}$$

We know for the spring-mass system that the period of oscillation is

$$T = 2\pi/\omega = 2\pi\sqrt{m/k} = 2\pi\sqrt{R/g} \tag{10}$$

after substituting in for k. For the package to travel to the other side of the world requires just half the period or

$$t = T/2 = \pi\sqrt{R/g} = \pi\left(\frac{6.37 \times 10^6\ m}{9.8\ m/s^2}\right)^{1/2} = 2532\ s = 0.704 \text{ hr}. \tag{11}$$

As this is simple harmonic motion, if the UPS person on the other side failed to catch the package, it would travel down the hole back to its starting point.