KEY

PHYS 1311 Spring 2017 Test 3
April 20, 2017

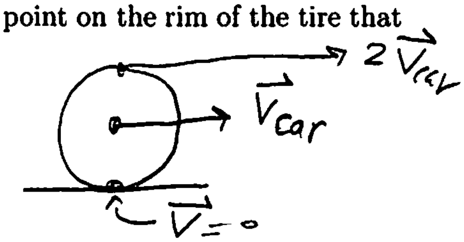
Name ______________ **Student ID** ______________ **Score** ______

Note: This test consists of one set of conceptual questions, five problems, and a bonus problem. For the problems, you *must show all* of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions when appropriate. An equation sheet is provided on the last pages.

Problem 1. Conceptual questions. State whether the following statements are *True* or *False.* (10 points total, no calculations required)

(a) When a tire rolls across the road without sliding, the point on the rim of the tire that touches the ground has zero instantaneous velocity.

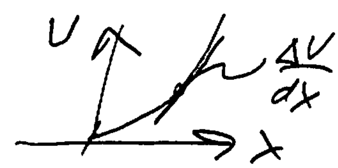
True



(b) In a plot of the potential energy versus position, the slope at a given position is equal to the negative of the force.

True

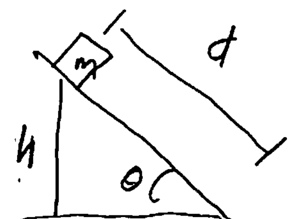
$$F_x = -\frac{dU}{dx}$$



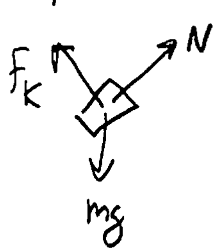
(c) The restoring force due to the spring is a dissipative force.

False $F_s = -kx$, it is conservative or non-dissipative

Problem 2. A 2.50×10^3 kg car is parked at the top of a sloped driveway when the breaks give out and the car coasts down the hill. If the driveway is sloped at 18.0° and the car travels 4.20 m, what is the speed of the car at the bottom, if there is a friction force of 3.50×10^3 N acting on the car? What is the work done by friction acting on the car? What is the final kinetic energy of the car? (15 points total)



$h = d\sin\theta$



- Using total energy update Equation $\Delta E = W_{surr}$
- Earth & Car are the system, driveway is the surroundings

$E = K + U = K + mgy$, $W = \vec{F}_K \cdot \vec{d}$

$= -F_K d = \boxed{-14700\,J}$

$E_f = E_i + W_{surr}$

$K_f + \overset{0}{U_f} = \overset{0}{K_i} + U_i - Fd$

$K_f = mgh - Fd = mgd\sin\theta - Fd$

$= [(2500)(9.8)(4.2)\sin 18° - (3500)(4.2)]$

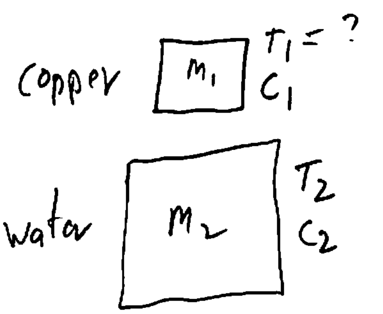
$= 31798 - 14700 = \boxed{17{,}100\,J}$

$V_f = \sqrt{\frac{2K_f}{m}}$

$= \sqrt{\frac{2(17{,}100)}{2500}}$

$= \boxed{3.70\,m/s}$

Problem 3. A block of copper of mass 0.25 kg is dropped into a thermos full of water originally at 5 °C. If the mass of the water is 3.0 kg and final temperature of the system is 10 °C, what was the original temperature of the copper block. The specific heats of copper and water are 3900 J/(kg °C) and 4200 J/(kg °C), respectively. Assume no heat is lost from the system. (15 points total)

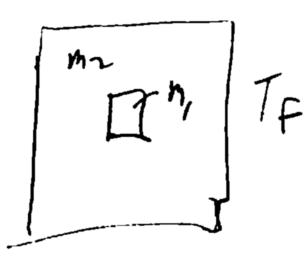


$\Delta E_{thermal} = mC\Delta T = \Delta E_{int}$

Total energy update $\Delta E = W_{surr} + Q = 0$

$K_1 + U_1 + K_2 + U_2 + E_{int,1} + E_{int,2} = 0$

$\Delta E = \Delta E_{int}(cu) + \Delta E_{int}(water) = 0$



$C_1 m_1 (T_1 - T_f) + C_2 m_2 (T_2 - T_f) = 0$

$T_1 - T_f = \frac{C_2 m_2}{C_1 m_1}(T_f - T_2)$

$T_1 = \frac{C_2 m_2}{C_1 m_1}(T_f - T_2) + T_f$

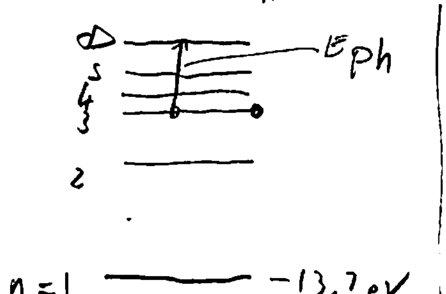
$= \frac{(4200)(3)}{(3900)(0.25)}[10 - 5] + 10 = \boxed{74.6\,°C}$

should have been 0.96 eV

Problem 4. What is the minimum energy in eV of a photon required to ionize the hydrogen atom initially in the $n = 3$ excited state? If instead, the photon only has 0.9742 eV, what is n of the final excited state after absorbing the photon? Explain what happened to the photon in either case. (15 points total)

The binding energies for the Hydrogen atom are given by

$E_n = -\frac{13.6}{n^2}$, $n = 1, 2, \ldots, \infty$



If the electron is in $n = 3$, and we want to ionize the atom, then $n = \infty$, So, the photon energy to ionize is

$E_{Ph} = -13.6\left(\frac{1}{\infty^2} - \frac{1}{3^2}\right) = \frac{13.6}{9} = \boxed{1.51\,eV}$

$0.9742 < 1.52\,eV$, so there is insufficient

Problem 5. A solid uniform sphere of of mass 22 kg ($=M$) and radius 0.70 m ($=R$) rolls across a horizontal surface. The speed of its center of mass is 4.0 m/s (V). (a) What is the rotational kinetic energy of the sphere? (b) What is the total kinetic energy of the sphere? (15 points total)

For a sphere $I = \frac{2}{5}MR^2$

$V_{sphere} = V_t = r\omega = V$

a) $K_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{V}{R}\right)^2$

$= \frac{1}{5}MR^2 = \frac{1}{5}(22)(4)^2 = \boxed{70.4\,J}$

b) $K = \frac{1}{2}MV^2 = \frac{1}{2}(22)(4)^2 = 176\,J$

$K_{Total} = K + K_{rot} = \boxed{246.4\,J}$

Energy to ionize. The electron ends up in a different n

$E_{Ph} = -13.6\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

$E_{Ph} - \frac{13.6}{n_i^2} = \frac{-13.6}{n_f^2}$

or $n_f = \sqrt{\frac{-13.6}{E_{Ph} - 13.6/n_i^2}}$

$= \sqrt{\frac{-13.6}{0.967 - 13.6/9}}$

$= \boxed{5.0 = n_f}$

Photon is destroyed.

Problem 6. Fun with derivations! (a) Starting with the momentum update equation in one-dimension, derive the work-kinetic energy theorem proving

$$\Delta K = W = \int F_x dx \tag{1}$$

where K is the translational kinetic energy for a particle with mass m and speed v being acting on by a force F_x that may not be constant with x (take $v << c$). (b) Starting with the relativistic version of the kinetic energy

$$K = (\gamma - 1)mc^2 \tag{2}$$

show that $K = \frac{1}{2}mv^2$ when $v << c$. (Hint: use the binomial expansion).

a) $p_f = p_i + F_x \Delta t$

$p_f - p_i = \Delta p = F_x \Delta t$

$$\frac{\Delta p}{\Delta t} = F_x$$

or $$\frac{dp}{dt} = \frac{d(mv)}{dt} = m\frac{dv}{dt} = F_x$$

chain rule

or $$m\frac{dv}{dx}\frac{dx}{dt} = m\frac{dv}{dx}v = F_x$$

or $m v\, dv = F_x dx$

Integrate from initial to final

$$\int_{v_i}^{v_f} mv\, dv = \frac{m}{2}\left[v\right]\Big|_{v_i}^{v_f} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

or $$K_f - K_i = \int_{x_i}^{x_f} F_x dx = W$$

$$\boxed{\Delta K = W}$$

b) $K = (\gamma - 1)mc^2$

where $\gamma = \left[1 - \left(\frac{v}{c}\right)^2\right]^{-1/2}$

apply binomial theorem

$(1 \pm \epsilon)^{-n} = 1 \mp n\epsilon + \cdots \quad \epsilon \ll 1$

$\epsilon = \left(\frac{v}{c}\right)^2, \; n = 1/2$

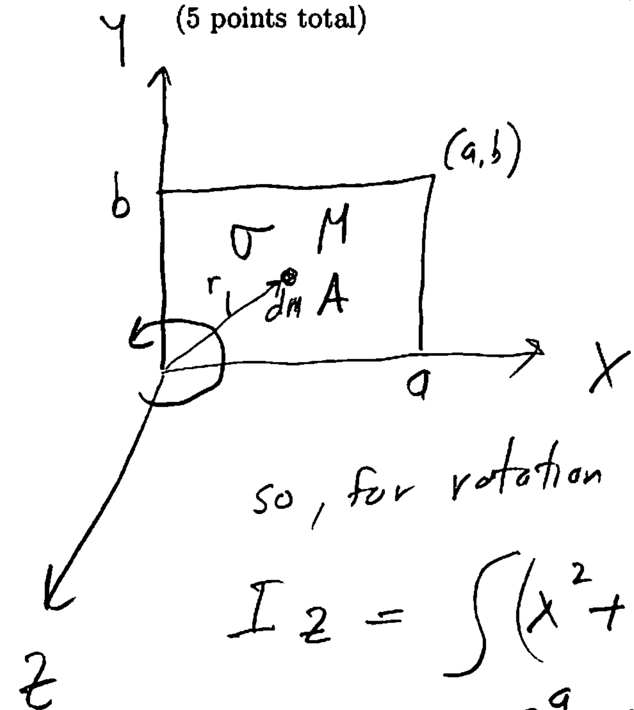
$\gamma = \left[1 + \left(\frac{1}{2}\right)\left(\frac{v}{c}\right)^2 + \cdots\right]$

So K becomes

$$K \simeq \left[\cancel{1} + \left(\frac{1}{2}\right)\left(\frac{v}{c}\right)^2 + \cdots \cancel{-1}\right]mc^2$$

$$= \frac{1}{2}\left(\frac{v}{c}\right)^2 mc^2 = \boxed{\frac{1}{2}mv^2 = K}$$

Bonus Problem. Consider a rectangular plate of mass M with horizontal length a and vertical length b in the $x-y$ plane (i.e., its corners are located at coordinates $< 0,0,0 >$, $< a,0,0 >$, $< a,b,0 >$, and $< 0,b,0 >$ m). If it has an areal density $\sigma = M/A$ kg/m^2, where A is the area of the plate, find the plates' moment of inertia for rotation about the z-axis. (5 points total)



$$I = \int r^2 dm$$

$$\sigma = \frac{M}{A} \Rightarrow M = \sigma A$$

or $dm = \sigma\, dx\, dy$ (in cartesian)

$$r^2 = x^2 + y^2$$

$A = ab$ or $\sigma = \frac{M}{ab}$

so, for rotation about the z-axis

$$I_z = \int (x^2+y^2)\,\sigma\, dx\, dy$$

$$= \sigma \int_{x=0}^{a}\int_{y=0}^{b} (x^2+y^2)\,\sigma\, dx\, dy$$

or

$$I_z = \sigma\left[\int_{x=0}^{a} x^2 dx \int_{y=0}^{b} dy + \int_{x=0}^{a} dx \int_{y=0}^{b} y^2 dy\right]$$

$$= \sigma\left[\frac{x^3}{3}\right]_0^a \left[y\right]_0^b + \sigma\left[x\right]_0^a \left[\frac{y^3}{3}\right]_0^b =$$

$$= \sigma\frac{a^3}{3} b + \sigma a\frac{b^3}{3} = \frac{\sigma}{3} ab\left[a^2+b^2\right] = \boxed{\frac{M}{3}\left[a^2+b^2\right]}$$