KEY

PHYS 1311 Spring 2017 Test 2
March 2, 2017

Name ____________________ **Student ID** ____________________ **Score** ______

Note: This test consists of one set of conceptual questions, five problems, and a bonus problem. For the problems, you *must show all* of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last page.

Problem 1. Conceptual questions. State whether the following statements are *True* or *False.* (10 points total, no calculations required)

(a) The period of oscillation for a simple pendulum is independent of its mass.

True $\omega = \frac{2\pi}{T} = \sqrt{\frac{g}{L}} \neq f(m)$

(b) The universal gravitation force is a good model to describe the interaction between atoms in a solid.

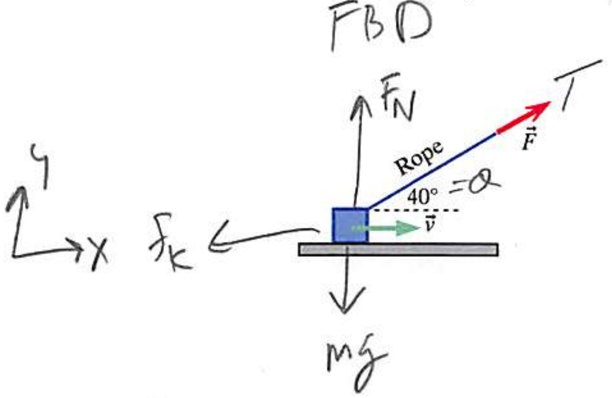
False, it is the Coulomb force modeled by springs

(c) When applying Newton's 2nd Law, or the momentum principle, the resulting net force acting on an object includes both internal and external forces.

False, $\vec{p}_f = \vec{p}_i + \vec{F}_{net}\Delta t$ ← external only

$m =$

Problem 2. You pull with a force of 225 N on a rope that is attached to a block of mass 30.0 kg, and the block slides across a rough floor at a constant speed of 1.1 m/s as in the figure. Show all forces acting on the block and find the magnitude of the normal force and the friction force. (15 points total)



$T\cos\theta = f_k$

$T\sin\theta = mg - F_N$

$f_k = (225N)\cos 40° = \boxed{172N}$

$F_N = mg - T\sin\theta$

$= (30)(9.8) - 225\sin 40°$

$= \boxed{149N}$

$\Sigma F_x = ma_x$

$T\cos\theta - f_k = ma_x = 0$

$\Sigma F_y = ma_y$

$T\sin\theta + F_N - mg = 0$

Problem 3. Evaluate the scalar product of the two vectors $\vec{A} = < 3, 4, 0 >$ and $\vec{B} = < 2, -6, 2 >$. What is the angle between the two vectors? (15 points total)

$\vec{A}\cdot\vec{B} = 3\cdot 2 + 4(-6) + 0(2) = -18$

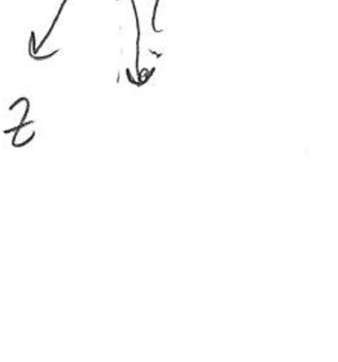
$A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{3^2 + 4^2 + 0} = 5$

$B = \sqrt{2^2 + 6^2 + 2^2} = \sqrt{44} = 6.6332$

From scalar product definition

$\vec{A}\cdot\vec{B} = AB\cos\phi \Rightarrow \phi = \cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right)$

$\phi = \cos^{-1}\left(\frac{-18}{5\cdot 6.6332}\right) = \boxed{122.9°}$



$\vec{V}_{ip}$

Problem 4. A proton with initial velocity of < 700, 0, 0 > km/s collides with an Fe atom (m_{Fe} = 56 u) which is at rest. After the collision, the proton's final velocity is < 600, 100, 0.00 > km/s, what is the recoil (final) velocity of the Fe atom? (15 points total)

$\vec{V}_{fp}$ =

Conservation of liner momentum $\quad m_p = m_0 = 1u, \quad m_{Fe} = 56u = 56 m_0$

$\vec{P}_i = \vec{P}_F$

$\vec{P}_{ip} + \vec{P}_{iFe} = \vec{P}_{fp} + \vec{P}_{fFe}$ (with $\vec{P}_{iFe} = 0$)

$m_0 \vec{V}_{ip} = m_0 \vec{V}_{fp} + 56 m_0 \vec{V}_{fFe}$

$56 \vec{V}_{fFe} = \vec{V}_{ip} - \vec{V}_{fp}$

$\vec{V}_{fFe} = \dfrac{\vec{V}_{ip} - \vec{V}_{fp}}{56} = \dfrac{1}{56}\left[\langle 700, 0, 0\rangle - \langle 600, 100, 0\rangle\right]$

$= \boxed{\langle 1.786, -1.786, 0\rangle \text{ km/s}}$

or $\vec{V} = 2.525$ km/s @ $-45°$

Problem 5. Consider the spring-mass system oscillating in the horizontal direction (neglect friction and air resistance) with mass m = 5.00 kg and force constant k = 200 N/m. If the system has a phase constant $\phi = \pi/4$ and has a speed of -2.00 m/s at t = 0, determine the amplitude A, maximum speed v_{max}, and maximum acceleration magnitude a_{max}. (15 points total)

SHM $\quad x(t) = A\cos(\omega t + \phi)$

$v(t) = -A\omega\sin(\omega t + \phi)$

$\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{200 \text{ N/m}}{5.00 \text{ kg}}} = \boxed{6.325 \text{ rad/s}}$

$t = 0$

$-2 \text{ m/s} = -A(6.325)\sin\dfrac{\pi}{4}$

or $\boxed{A = 0.4472 \text{ m}}$

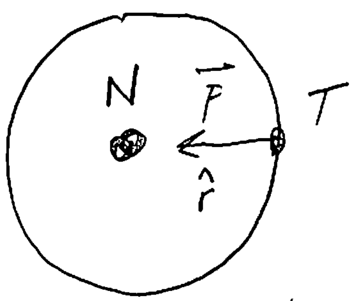
$V_{max} = A\omega = (0.4472 \text{ m})(6.325 \text{ rad/s}) = \boxed{2.83 \text{ m/s}}$

$a_{max} = A\omega^2 = (0.4472)(6.325 \text{ rad})^2 = \boxed{17.9 \text{ m/s}}$

Problem 6. (a) Beginning with Newton's 2nd Law and the universal gravitational force, derive a relation for the orbital speed of the moon Triton about Neptune assuming the mass of Neptune is much larger than that of Triton. (b) Derive a relation for the orbital period. (c) From (a) and (b) determine numerically the mass of Neptune and the orbital speed of Triton, if its distance from Neptune is 353×10^3 km and its orbital period is 5.9 days. (30 points total)

" r " T

a)



For uniform circular orbit

$$\Sigma F_r = m_T a_r$$

$$G\frac{M_N m_T}{r^2} = m_T \frac{V^2}{r}$$

$$V = \sqrt{\frac{GM_N}{r}}$$

b) $T = \frac{2\pi r}{V} = \frac{\text{circumference}}{\text{orbital speed}}$

$$= 2\pi r\sqrt{\frac{r}{GM_N}} = \frac{2\pi r^{3/2}}{\sqrt{GM_N}} = T$$

or $T^2 = \frac{4\pi^2 r^3}{GM_N}$

c) or $M_N = \frac{4\pi^2 r^3}{GT^2}$

$$= \frac{4\pi^2 (353\times10^6\ m)^3}{(6.67\times10^{-11})(509{,}760)^2}$$

$$= 1.00\times10^{26}\ kg$$

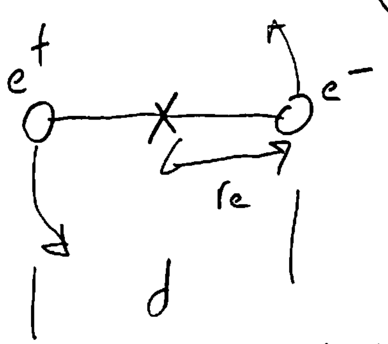
$$T = 5.9\ \text{days} \times \frac{24\ \text{hours}}{1\ \text{day}} \times \frac{60\ \text{min}}{1\ \text{hr}} \times \frac{60\ s}{1\ \text{min}}$$

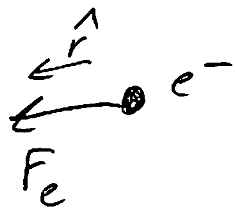
$$= 509{,}760\ s$$

$$V = \sqrt{\frac{(6.67\times10^{-11})(1\times10^{26}\ kg)}{353\times10^6\ m}} = 4347\ m/s = 4.35\ km/s$$

Bonus Problem. Consider the positronium atom, a hydrogen-like atom in which the proton is replaced by a positron. A positron has the same mass as an electron, but opposite charge. Consider that the two particles orbit each only, but only interact through the Coulomb force, derive relations for their orbital speeds and orbital periods. Take the distance between the two particles to be 2×10^{-10} m and compute the speeds and periods. (Hint: they will both have the same speed and period). (5 points total)

d





$d = 2r_e$, $q_e = -e$, $q_p = e$

Coulomb force acts between particles and is attractive.

Consider electron

$$\Sigma F_e = m_e a_e$$

$$\frac{-k_e q_e q_p}{d^2} = m_e \frac{v_e^2}{r_e}$$

$$\frac{-k_e(-e)(e)}{d^2 m_e} = \frac{v_e^2}{d/2}$$

$$\boxed{v_e = \sqrt{\frac{k_e e^2}{2 d m_e}}} = \sqrt{\frac{(8.99\times10^9)(1.602\times10^{-19})^2}{2(2\times10^{-10})(9.109\times10^{-31})}}$$

$$= \boxed{7.96\times10^5 \text{ m/s}}$$

$$T = \frac{2\pi r_e}{v_e} = \frac{2\pi\, d/2}{\sqrt{\frac{k_e e^2}{2 d m_e}}} = \frac{\pi d^{3/2}}{e}\sqrt{\frac{2 m_e}{k_e}}$$

$$T = \frac{2\pi r_e}{v_e} = \frac{2\pi(1\times10^{-10})}{7.96\times10^5} = \boxed{7.896\times10^{-16}\text{ s}}$$