PHYS 1311: In Class Problems
Chapter 1 Solutions
Jan. 14, 2016

Problem 1. Derive in 1D the velocity update equation from the definition of acceleration, $a_x = dv_x/dt$ and the position update equation from the definition of velocity $v_x = dx/dt$.

Solution

Rewrite the acceleration equation as $dv_x = a_x dt$, which is an equation for the differentials. This can be recast as an integral relation with limits from t_i to t_f and v_{xi} to v_{xf}, where i and f refer to initial and final. This gives

$$\int_{v_{xi}}^{v_{xf}} dv_x = \int_{t_i}^{t_f} a_x dt. \tag{1}$$

Carrying out the integrals, but for the case of constant a_x gives

$$v_x\Big|_{v_{xi}}^{v_{xf}} = a_x t\Big|_{t_i}^{t_f}, \tag{2}$$

or

$$v_{xf} - v_{xi} = a_x(t_f - t_i), \tag{3}$$

to give the velocity update equation

$$v_{xf} = v_{xi} + a_x(t_f - t_i). \tag{4}$$

Here v_{xf} and t_f are variabiles. Now for the position update equation, rewrite the velocity relation as $dx = v_x dt$. Take v_x to be v_{xf}, t_f to be t, and substitute in equation (4) to give

$$\int_{x_i}^{x_f} dx = \int_{t_i}^{t_f} v_x dt = \int_{t_i}^{t_f} [v_{xi} + a_x(t - t_i)]dt. \tag{5}$$

Carry out the integrals on the left and right with initial and final positions x_i and x_f gives

$$x\Big|_{x_i}^{x_f} = v_{xi}t\Big|_{t_i}^{t_f} + \frac{1}{2}a_x t^2\Big|_{t_i}^{t_f} - a_x t_i t\Big|_{t_i}^{t_f} \tag{6}$$

since v_{xi} and t_i are constants. This gives after some algebra

$$x_f = x_i + v_{xi}(t_f - t_i) + \frac{1}{2}a_x(t_f - t_i)^2, \tag{7}$$

which is one form of the position update equation for a particle with a constant acceleration.

Problem 2. Speculate (without reading ahead) why the concept of momentum $\vec{p} \approx m\vec{v}$ is introduced (or why velocity ain't good enough).

Solution

It is clear that the momentum is a vector and merely obtained by multiplying the velocity by a scalar (mass). There is many reasons why this quantity is of interest. For example, it will be key to our use of Newton's Second Law in Chapter 2. However, more importantly, it is one property of a system of particles which may (and usually does) remains constant as it evolves in time. Such a property is said to be a "constant of the motion" and may define a conservation principle. Velocity does not have any of these properties.