Kinematics in 1D

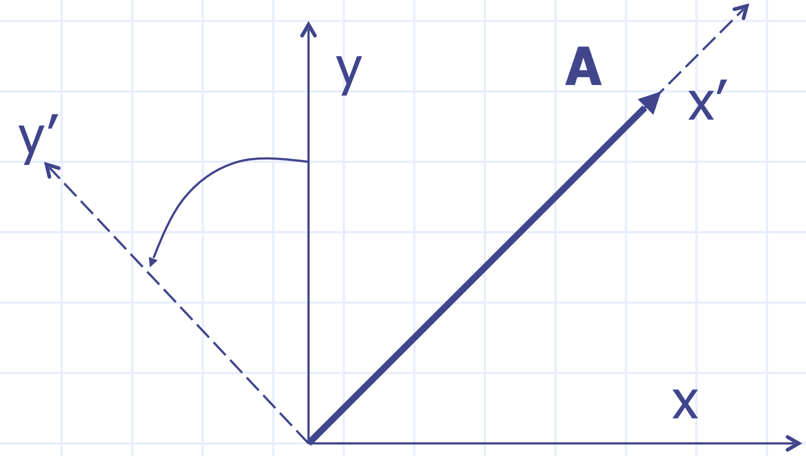
- **Mechanics** - the study of the motion of objects (atoms, blood flow, ice skaters, cars, planes, galaxies, …)

- Kinematics - describes the motion of an object without reference to the cause of the motion

- Dynamics - describes the effects that forces have on the motion of objects (Chapter 5)

- [Statics - describes the effects that forces have on an object which is at rest (bridge, building, ….)]

- Kinematics provides answers to the questions: 1) Where is an object? 2) What is its velocity? 3) What is its acceleration?



Displacement

$\Delta \mathbf{x} = \mathbf{x_f} - \mathbf{x_i}$ = displacement [L]

= 15 m - 5 m = 10 m in positive x-direction

If $\mathbf{x_f}$=-5 m, then

$\Delta \mathbf{x}$ = -5 m - 5 m = -10 m in positive x-direction

or (10 m in the negative x-direction)

Speed and Velocity

Average speed = distance/(elapsed time)=D/Δ t

While average velocity is displacement/(elapsed time):

$$v_{x,avg} = \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t} \quad \frac{[L]}{[T]}$$

t = time (sec),

time is a scalar

in 1-dimension. Or in vector notation:

$$\vec{v}_{avg} = \frac{\vec{x}_f - \vec{x}_i}{t_f - t_i} = \frac{\Delta\vec{x}}{\Delta t}$$

A vector with dimensions of [L]/[T], in S.I., units are m/s

Simple Example

A traveler arrives late at the airport at 1:08pm. Her plane is scheduled to depart at 1:22pm and the gate is 2.1 km away. What must be her minimum average running speed (in m/s) to make the flight?

Solution

Given: t_i = 1:08 pm, t_f = 1:22 pm, D = 2.1 km

What is average speed v_{av}?

$\Delta t = t_f - t_i$ = 1:22 - 1:08 = 14 mins

$v_{av} = D/\Delta t$ = (2.1 km)/(14 mins) = 0.15 km/min

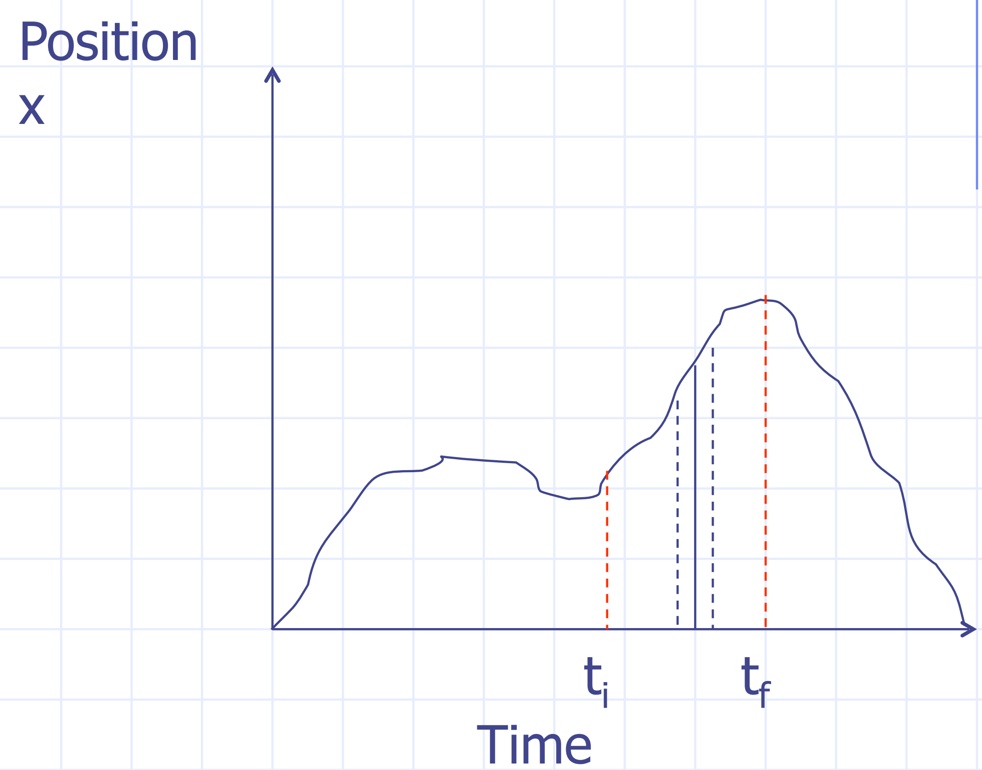
= (0.15 km/min)(1000 m/1 km)(1 min/ 60 s)

v_{av}=2.5 m/s

Instantaneous Speed and Velocity

- Instantaneous velocity is the velocity at some instant in time (as Δt goes to zero).

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$



- Instantaneous speed is the magnitude of the instantaneous velocity.

- Instantaneous velocity in vector notation

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$$

Acceleration

The change in (instantaneous) velocity of an object, gives the average acceleration:

$$a_{x,avg} = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{\Delta v_x}{\Delta t}$$

[L]/[T^2]

In S.I., units are m/s^2

Instantaneous acceleration

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$

$$= \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2 x}{dt^2}$$

We will mostly consider constant accelerations

Equations of Kinematics

- Starting with the definitions of displacement, velocity, and acceleration, we can derive equations that allow us to predict the motion of an object
- Here we consider constant acceleration
- Acceleration:

$$a_{x,avg} = \frac{\mathrm{v}_{xf} - \mathrm{v}_{xi}}{t_f - t_i} = a_x$$

Solve for v

$$a_x(t_f - t_i) = \mathrm{v}_{xf} - \mathrm{v}_{xi}$$

$$\boxed{\mathrm{v}_{xf} = \mathrm{v}_{xi} + a_x(t_f - t_i)}$$

Velocity update equation

- Velocity:

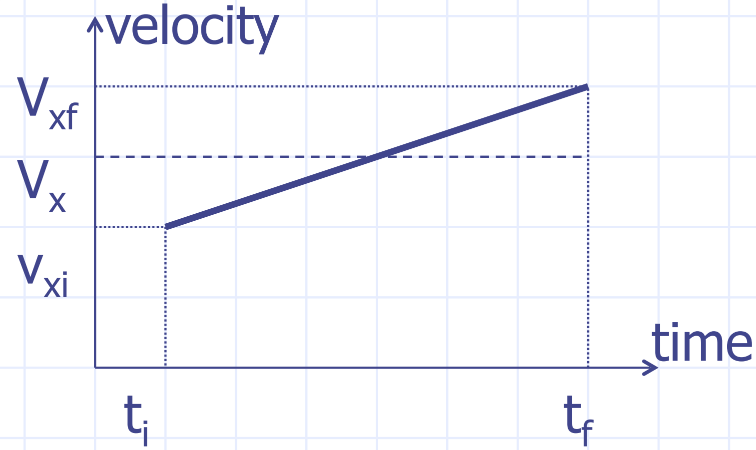
$$\mathrm{v}_{x,avg} = \frac{x_f - x_i}{t_f - t_i}$$

Solve for distance

$$\mathrm{v}_{x,avg}(t_f - t_i) = x_f - x_i$$

$$x_f = x_i + \mathrm{v}_{x,avg}(t_f - t_i)$$

Position update equation



What is average velocity? If acceleration is constant, the average velocity is the mean of the initial and final velocity:

$$\mathrm{v}_{x,avg} = \frac{1}{2}(\mathrm{v}_{xi} + \mathrm{v}_{xf})$$

$$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})(t_f - t_i)$$

Also, can substitute in the velocity v to give:

$$x_f = x_i + \frac{1}{2}[v_{xi} + v_{xi} + a_x(t_f - t_i)](t_f - t_i)$$

Or:

$$x_f = x_i + v_{xi}(t_f - t_i) + \frac{1}{2}a_x(t_f - t_i)^2$$

What if we have no information about time?

It can be removed from the equations.

From acceleration equation:

$$t_f - t_i = \frac{v_{xf} - v_{xi}}{a_x}$$

Substitute into x equation

$$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})\frac{(v_{xf} - v_{xi})}{a_x}$$

$$x_f = x_i + \frac{(v_{xf}^2 - v_{xi}^2)}{2a_x}$$

Since $(v_{xi} + v_{xf})(v_{xf} - v_{xi}) = (v_{xf}^2 - v_{xi}^2)$

Then, solving for v_{xf} gives:

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

Summary – Equations of Kinematics

$$x_f = x_i + \frac{1}{2}(\mathrm{v}_{xi} + \mathrm{v}_{xf})(t_f - t_i)$$

$$x_f = x_i + \mathrm{v}_{xi}(t_f - t_i) + \frac{1}{2}a_x(t_f - t_i)^2$$

$$\mathrm{v}_{xf} = \mathrm{v}_{xi} + a_x(t_f - t_i)$$

$$\mathrm{v}_{xf}^2 = \mathrm{v}_{xi}^2 + 2a_x(x_f - x_i)$$

Example Problem

A car is traveling on a dry road with a velocity of +32.0 m/s. The driver slams on the brakes and skids to a halt with an acceleration of –8.00 m/s^2. On an icy road, the car would have skidded to a halt with an acceleration of –3.00 m/s^2. How much further would the car have skidded on the icy road compared to the dry road?

<u>Solution:</u>

Given: $\mathbf{v}_i$ = 32.0 m/s in positive x-direction

$\mathbf{a}_{dry}$ = -8.00 m/s^2 in positive x-direction

$\mathbf{a}_{icy}$ = -3.00 m/s^2 in positive x-direction

Also, $v_f=0$, assume $t_i=0$, $x_i=0$

Find x_{dry} and x_{icy}, or $x_{icy}-x_{dry}$

Example Problem

A Boeing 747 Jumbo Jet has a length of 59.7 m. The runway on which the plane lands intersects another runway. The width of the intersection is 25.0 m. The plane accelerates through the intersection at a rate of -5.70 m/s^2 and clears it with a final speed of 45.0 m/s. How much time is needed for the plane to clear the intersection?

<u>Solution:</u>

Given: $\mathbf{a}$ = -5.70 m/s^2 in x-direction

$\mathbf{v}_f$ = +45.0 m/s in x-direction

L_{plane} = 59.7 m, $L_{intersection}$ = 25.0 m

Assume, t_i = 0 when nose of Jet enters intersection.

Find t_f when tail of Jet clears intersection.

Example Problem (you do)

An electron with an initial speed of 1.0×10^4 m/s enters the acceleration grid of a TV picture tube with a width of 1.0 cm. It exits the grid with a speed of 4.0×10^6 m/s. What is the acceleration of the electron while in the grid and how long does it take for the electron to cross the grid?

Solution:

Given: $\mathbf{v}_i = +1.0 \times 10^4$ m/s in x-direction

$\mathbf{v}_f = +4.0 \times 10^6$ m/s in x-direction

$\Delta x = 1.0$ cm

Find a ($= 8.0 \times 10^{14}$ m/s^2) and t_f ($= 5.0$ ns).

Motion in Free-fall

- Consider 1D vertical motion on the surface of a very massive object (Earth, other planets, the sun, even large asteroids)
- Replace x with y in 1D kinematic equations
- Acceleration is always non-zero (but constant)
- Acceleration of an object is due to gravity (we will study gravitational forces later)
- All objects near the surface of the Earth experience the same constant, downward acceleration

- The acceleration due to gravity does not depend on the mass, size, shape, density, or any intrinsic property of the falling object
- The acceleration due to gravity does not depend on height (for heights near the Earth's surface)
- For Earth, the acceleration due to gravity has the value (notice g is the magnitude of the acceleration, i.e., a scalar, therefore positive):

$$g = 9.80\ m/s^2 \text{ or } 32.2\ ft/s^2$$

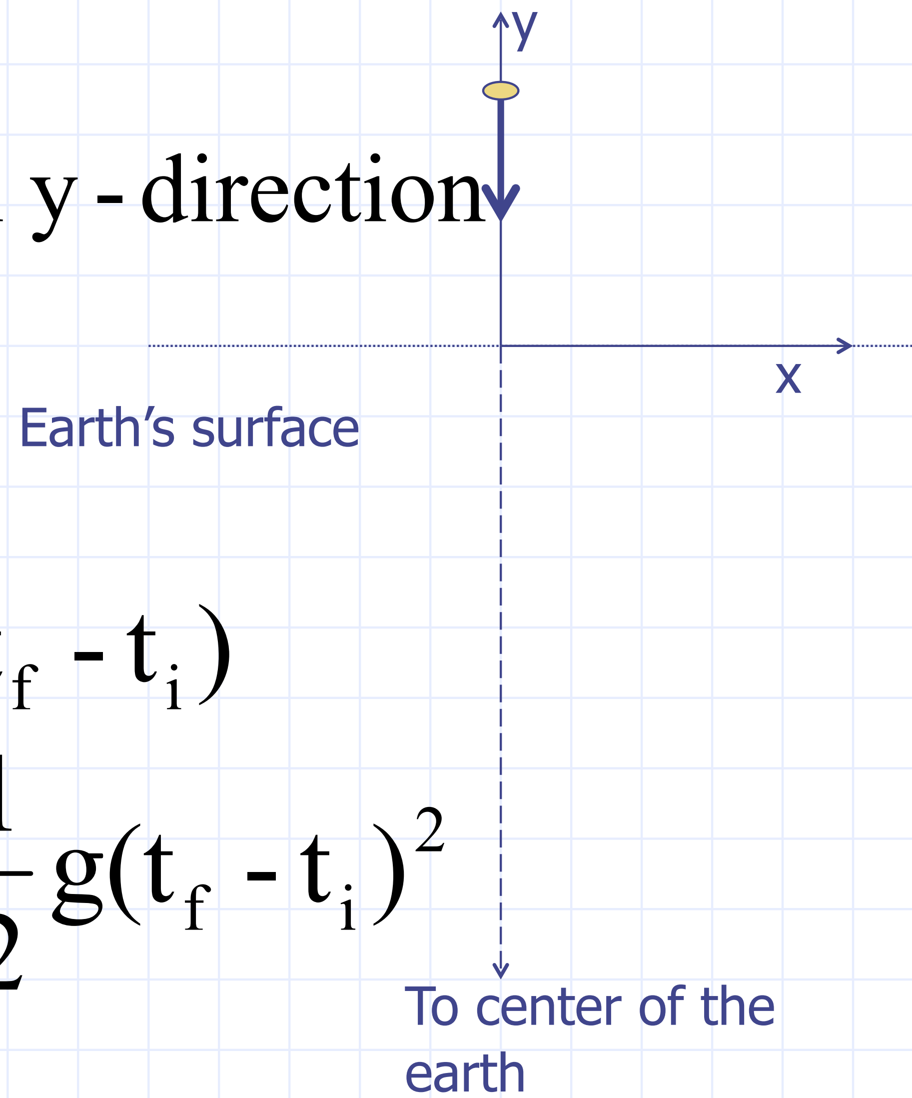
- For other bodies, g has different values:

$$g_{moon} = 1.60\ m/s^2$$

$$g_{Jupiter} = 26.4\ m/s^2$$

Taipei 101

$$\vec{a}_y = -g \text{ in } y\text{-direction}$$



Kinematic Equations

$$y_f = y_i + \frac{1}{2}(v_{yi} + v_{yf})(t_f - t_i)$$

$$y_f = y_i + v_{yi}(t_f - t_i) - \frac{1}{2}g(t_f - t_i)^2$$

$$v_{yf} = v_{yi} - g(t_f - t_i)$$

$$v_{yf}^2 = v_{yi}^2 - 2g(y_f - y_i)$$

Example Problem

A ball is thrown upward from the top of a 25.0-m tall building. The ball's initial speed is 12.0 m/s. At the same instant, a person is running on the ground at a distance of 31.0 m from the building. What must be the average speed of the person if he is to catch the ball at the bottom of the building?

Solution: Two particles, one with x-motion, one with y-motion

Given: $\mathbf{v}_{yib}$ = +12.0 m/s in y-direction

y_{ib}=25.0 m, x_{ip}=31.0 m

What is average speed of runner?