PHYS 1311 Chapter 6

extra not for grading

3-30-2016

Problems P13, P25, P29, P31, P39, ~~P42~~, ~~P45~~, ~~P48~~, P51, P59, ~~P62~~, P69, ~~P71~~, P73, P75

Problem P13

^4He, $v = 0.9993\,c$

$m_p = 6.4\times10^{-27}\,kg$

a) $E_{rest} = mc^2 = (6.4\times10^{-27}\,kg)(3\times10^8\,m/s)^2 = \boxed{5.76\times10^{-10}\,J} = 3.596\ GeV$

b) can not use $K = \frac{1}{2}mv^2$

c) $K = (\gamma - 1)\,mc^2 = (\gamma-1)\,E_{rest}$

$$\gamma = \frac{1}{\sqrt{1-0.9993^2}} = 26.7308$$

$$K = (26.7308 - 1)\,E_{rest} = \boxed{1.48\times10^{-8}\,J}$$

$= 92.5\ GeV$

d) $\boxed{B}$ is true, kinetic energy much greater than rest energy $K \gg E_{rest}$

Problem P25

$\vec{F} = \langle 23, -12, 32\rangle\,N$

$\Delta\vec{r} = \langle 0.12, 0.31, -0.24\rangle\,m$

$$W = \vec{F}\cdot\Delta\vec{r} = F_x\Delta x + F_y\Delta y + F_z\Delta z$$
$$= (23)(0.12) + (-12)(0.31) + (32)(-0.24) = \boxed{-8.64\,J}$$

3-30-2016

Problem P29

$\vec{F}_1 = \langle 34, 0, 0\rangle N$

$\vec{r}_1 = \langle 2, 0, 0\rangle m$

$\vec{F}_2 = \langle 13, 0, 0\rangle N$, $\vec{F}_3 = \langle -40, 0, 0\rangle N$

$\vec{r}_2 = \langle 6, 0, 0\rangle m$, $\Delta\vec{r}_3 = \langle 2, 0, 0\rangle m$

$$W = W_1 + W_2 = \vec{F}_1 \cdot \vec{r}_1 + \vec{F}_2 \cdot \vec{r}_2 + \vec{F}_3 \cdot \vec{r}_3$$
$$= (34)(2) + (13)(6) + (-40)(2)$$
$$= 68 + 78 - 80 = \boxed{66\ J}$$

Problem P31

$m_{Li} = 5.1 \times 10^{-27} kg$, $V = 0.984c$

$$E_{rest} = mc^2 = (5.1 \times 10^{-27})(3 \times 10^8)^2 = \boxed{4.59 \times 10^{-10} J}$$

$$\gamma = \frac{1}{\sqrt{1 - .984^2}} = 5.6127$$

$$E = \gamma E_{rest} = (5.6127)(4.59 \times 10^{-10} J) = \boxed{2.58 \times 10^{-9} J}$$

$$K = E - E_{rest} = (\gamma - 1) E_{rest} = \boxed{2.11 \times 10^{-9} J}$$

$$W = K_f - K_i \Rightarrow K_f = W + K_i = 4.7 \times 10^{-9} + 2.11 \times 10^{-9}$$
$$= 4.7 \times 10^{-9} J \qquad = \boxed{6.81 \times 10^{-9} J}$$

$$E_{rest\,i} = E_{rest\,f} = \boxed{4.59 \times 10^{-10} J}$$

$$E_f = E_i + W = 2.58 \times 10^{-9} + 4.7 \times 10^{-9} = \boxed{7.28 \times 10^{-9} J}$$

3-30-2016

Problem P39 $\quad m_e, \quad v_i = 0.99c, \quad v_f = 0.93c, \quad \Delta x = 3m$

$$\Delta E = W_{surr} = \vec{F}\cdot\Delta\vec{r} = \vec{F}\cdot\Delta\vec{x} \qquad \gamma_f = \frac{1}{\sqrt{1-0.93^2}} = 2.72$$

$$\gamma_f m_e c^2 - \gamma_i m_e c^2 = F\Delta X$$

$$m_e c^2(\gamma_f - \gamma_i) = F\Delta x$$

$$F = \frac{m_e c^2(\gamma_f - \gamma_i)}{\Delta x} \qquad \gamma_i = \frac{1}{\sqrt{1-0.99^2}} = 7.089$$

$$= \frac{(9\times10^{-31})(2.72 - 7.089)(3\times10^8)^2}{3}$$

$$= -1.18\times10^{-13} N \rightarrow |\vec{F}| = \boxed{1.2\times10^{-13} N}$$

Problem P42 $\quad m_e, \quad \Delta X = 2\ mi, \quad v_i = 0 \Rightarrow \gamma_i = 1$

$F_x = 2\times10^{-12} N$

$\Delta x = 3200 m$

$$\Delta E = W_{surr}$$

$$\gamma_f m_e c^2 - \gamma_i m_e c^2 = F\Delta X$$

or $$\gamma_f = \gamma_i + \frac{F\Delta X}{m_e c^2} = 1 + \frac{(2\times10^{-12} N)(3200 m)}{(9.109\times10^{-31} kg)(2.998\times10^8 \frac{m}{s})^2}$$

$$= 1 + 78{,}171$$

$$E_f = \gamma m_e c^2 = (78{,}172)(9.109\times10^{-31} kg)(2.998\times10^8 \tfrac{m}{s})^2$$

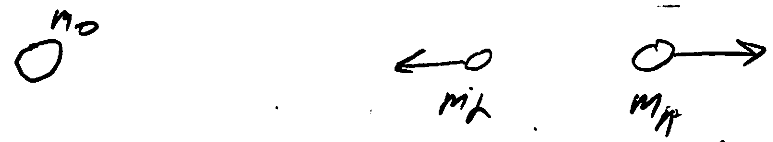
$$= \boxed{6.4\times10^{-9} J} = 39.95\ GeV$$

$$E_{rest} = \frac{E_f}{\gamma} = 8.187\times10^{-14} J = 0.511\ MeV$$

$\vec{p} = \gamma_f m \vec{v}_f$ or use $E^2 - p c^2 = (m_e c^2)^2$ [bottom of next page] initial 3-30-2006

Problem P 45 $\quad m_o = 3.874028 \times 10^{-25}$ kg , $V_i = 0$, $\gamma_i = 1$

final $\; m_\alpha = 6.640678 \times 10^{-27}$ kg , $m_n = 3.827555 \times 10^{-25}$ kg



a) $E^o_{rest} = m_o c^2 = (3.894028 \times 10^{-25})(2.99792 \times 10^8 \, m/s)$

$= \boxed{3.49976714 \times 10^{-8} \, J}$

$E_{final,\,rest} = (m_\alpha + m_n) c^2 = (6.640678 \times 10^{-27} + 3.827555 \times 10^{-25})(2.99792 \times 10^8)$

$= \boxed{3.49970762 \times 10^{-8} \, J}$

c) Rest energy $\boxed{\text{decreased}}$

$E_f = E_i$

$m_\alpha c^2 + m_n c^2 + K_{f\alpha} + K_{fn} = m_o c^2$

$K_{f\alpha} + K_{fn} = m_o c^2 - (m_\alpha c^2 + m_n c^2) = 5.952 \times 10^{-13} \, J$

$= \boxed{3.715 \text{ MeV}}$

Problem P42 (cont'd) $\quad$ Since $E \gg m_e c^2$

$p_f = \dfrac{E_f}{c} = \dfrac{6.4 \times 10^{-9} \, J}{2.998 \times 10^8 \, m/s} = \boxed{2.13 \times 10^{-17} \, kg \, \frac{m}{s}}$

$= 39.95 \text{ GeV}/c$

3-30-2006

Problem P42 (cont'd)

$$\gamma_f = \left[1-\left(\frac{v}{c}\right)^2\right]^{-1/2} \rightarrow v_f = c\left[1-\frac{1}{\gamma_f^2}\right]^{1/2} \simeq c$$

so use binomial expansion

$$v_f \simeq c\left[1-\frac{1}{2}\frac{1}{\gamma_f^2}+\cdots\right] = c\left[1-\frac{1}{2(78{,}172)^2}\right]$$

$$= c\left[1-8{,}182\times10^{-11}\right] \simeq \boxed{c}$$

b) $\Delta x = \left(\frac{v_f+v_i}{2}\right)\Delta t = \frac{v_f}{2}\Delta t \simeq \frac{c}{2}\Delta t$

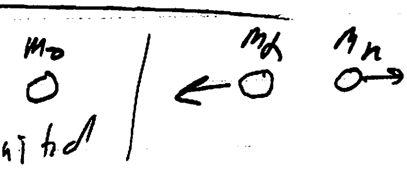
$$\Delta t = \frac{2\Delta x}{c} = \frac{2(3200\,m)}{2.998\times10^8\,m/s} = \boxed{2{,}13\times10^{-5}\,s}$$

Problem P48

$m_0 = 3.917268\times10^{-25}\,kg$

$m_\alpha = 6.640678\times10^{-27}\,kg$

$m_n = 3.850768\times10^{-25}\,kg$



$c = 2.99792\times10^8\,m/s$

$E_f = E_i$

$m_\alpha c^2 + K_\alpha + m_n c^2 + K_n = m_0 c^2$

a) $K_\alpha + K_n = (m_0 - m_\alpha - m_n)c^2 = (9.322\times10^{-30})(\ \ )^2$

$= \boxed{8{,}3782\times10^{-13}\,J} = 5.23\ MeV$

$\ll E_{rest,\alpha}, E_{rest,n}$

b) $\vec{P}_i = \vec{P}_f$

$0 = -\gamma_\alpha m_\alpha v_\alpha + \gamma_n m_n v_n$

$E_{rest,\alpha} = 3.73\,GeV$

$E_{rest,n} = 216\,GeV$

3-30-2016

Problem P48 (con't'd)

assume that $\gamma_\alpha \sim \gamma_n \sim 1$ since $K_\alpha + K_n \ll E_{rest}$

$$m_\alpha v_\alpha = m_n v_n \qquad \frac{1}{2} m_\alpha v_\alpha^2 + \frac{1}{2} m_n v_n^2 = K_{total}$$

$$v_\alpha = \frac{m_n}{m_\alpha} v_n \rightarrow \frac{1}{2} m_\alpha \frac{m_n^2}{m_\alpha^2} v_n^2 + \frac{1}{2} m_n v_n^2 = K_{total}$$

or $$v_n^2 \frac{m_n}{2}\left[\frac{m_n}{m_\alpha} + 1\right] = K_{total}$$

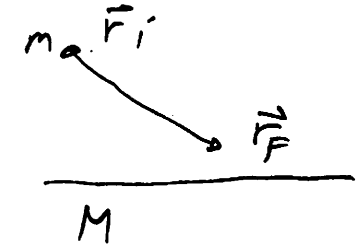
or $$\frac{1}{2} m_n v_n^2 = K_n = \frac{K_{total}}{\left[\frac{m_n}{m_\alpha} + 1\right]} = \frac{8.3782 \times 10^{-13}\ \text{J}}{\left[\frac{3.851 \times 10^{-25}}{6.64 \times 10^{-27}} + 1\right]}$$

$$= \boxed{1.4 \times 10^{-14}\ \text{J}}$$

$$= 0.089\ \text{MeV}$$

$$K_\alpha = K_{total} - K_n = \boxed{8.24 \times 10^{-13}\ \text{J}} = \boxed{5.14\ \text{MeV}}$$

Problem P51



$$\Delta U = U_f - U_i$$

$$= mg y_f - mg y_i$$

$$= mg(y_f - y_i)$$

$$\vec{r}_i = \langle 22, 43, -41 \rangle \text{m}$$

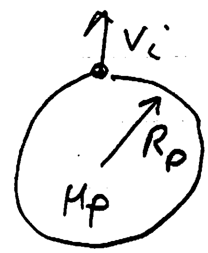
$$\vec{r}_f = \langle -27, 11, 46 \rangle \text{m}$$

$$= (7\ \text{kg})(9.8\ \tfrac{\text{m}}{\text{s}^2})(11 - 43)\text{m}$$

$$= \boxed{-2200\ \text{J}}$$

3-30-2010

Problem P59



$R_p = 2\times10^6 m$, $M_p = 1.2\times10^{23} kg$

$V_f = 900 m/s$, $r_p \to \infty$

a) $V_i = ?$

$$E_f = E_i + \cancel{W_{surr}}^{\,0}$$

$$\tfrac{1}{2}\cancel{m}V_f^2 - \underbrace{\frac{GM_p\cancel{m}}{r_f}}_{0} = \tfrac{1}{2}\cancel{m}V_i^2 - \frac{GM_p\cancel{m}}{R_p^2}$$

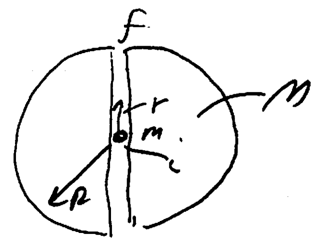
$$V_i = \sqrt{V_f^2 + 2\frac{GM_p}{R_p}} = \sqrt{900^2 + \frac{2(6.67\times10^{-11})(1.2\times10^{23})}{(2\times10^6)}}$$

$$= \boxed{2969 \ m/s}$$

b) For escape speed $V_f = 0$

$$V_i = V_{esc} = \sqrt{\frac{2GM_p}{R_p}} = \boxed{2829 \ m/s}$$

Problem P67



a) Inside earth $g(R) = g\frac{r}{R} = \frac{F}{M}$

$$W = -\Delta U = \int_{r_i}^{r_f} \vec{F}\cdot d\vec{r} = \int \frac{gr}{R} m\, dr$$

$$= \frac{gm}{R}\int_{r_i}^{r_f} r\,dr = \frac{gm}{R}\left[\frac{r^2}{2}\right]_0^R = \boxed{\frac{gmR}{2}}$$

3-30-2016

Problem P67

$$r_i = R, \quad r_f = \infty$$

$$W = +\int_R^{\infty} \frac{GMm}{r^2}\,dr = +GMm\int_R^{\infty}\frac{dr}{r^2}$$

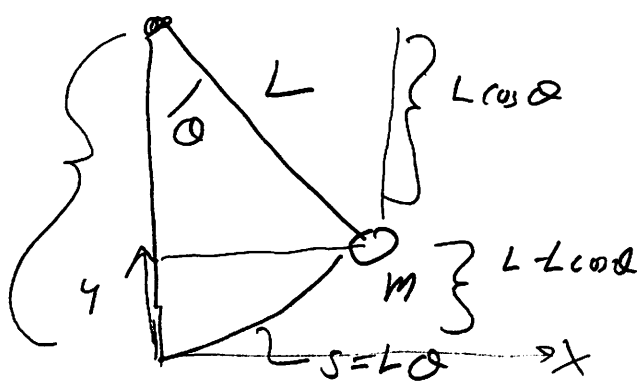
$$= -GMm \left.\frac{1}{r}\right|_R^{\infty} = -GMm\left[0 - \frac{1}{R}\right] = \frac{GMm}{R}$$

$$= \left(\frac{GM}{R^2}\right) mR = gmR =$$ 2 (result in part a)

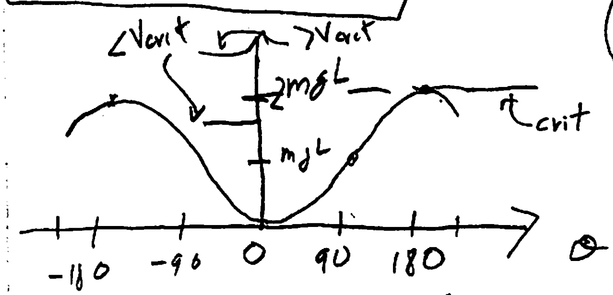
Problem P69

a) $U = mgy$

$$\boxed{U = mgL(1-\cos\theta)}$$



b)



c) $$F_s = -\frac{dU}{ds} = -\frac{d}{ds}\left[mgL\left(1-\cos\left(\frac{s}{L}\right)\right)\right] = mgL\frac{d}{ds}\left[\cos\left(\frac{s}{L}\right)\right]$$

$$= -mgL\frac{1}{L}\sin\frac{s}{L} = \boxed{-mg\sin\theta}$$

3-30-2016

Problem P69 (cont'd) $V_i \neq 0$, $\theta_i = 0$, $\theta_f = 180^\circ$, $V_f = 0$

$E_f = E_i + \cancel{W_{surr}}^{\,0}$

$mgy_f + \frac{1}{2}m\cancel{V_f^2}^{\,0} = mg\cancel{y_i}^{\,0} + \frac{1}{2}mV_i^2$

$\cancel{m}gL(1-\cos\theta_f) = \frac{1}{2}\cancel{m}V_i^2$

$V_i = \sqrt{2gL(1-\cos\theta_f)} = \sqrt{2gL(1-\cos\pi)}$

$\boxed{V_i = 2\sqrt{gL}}$

Problem P71

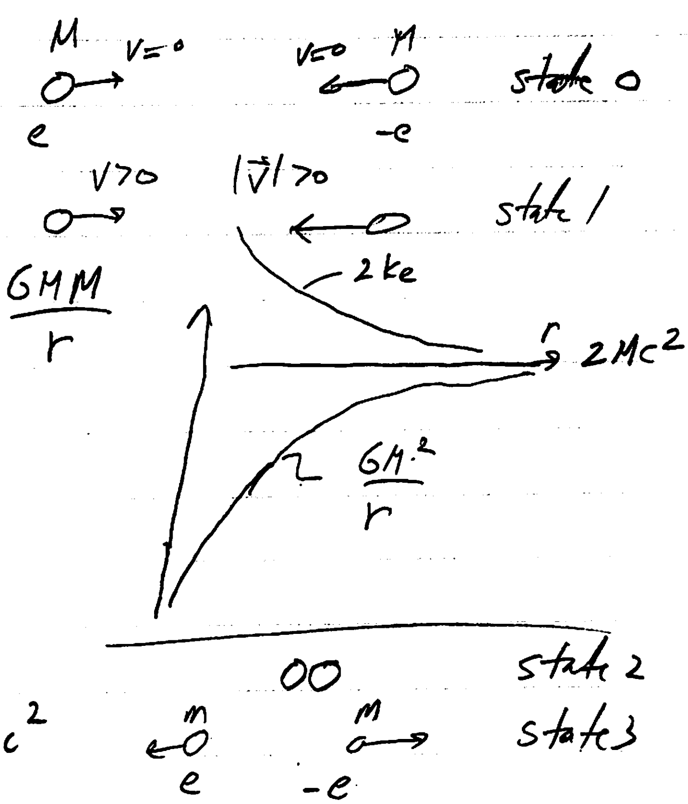
a. Total energy

$E_e + E_{-e}$

$\gamma_e M_e c^2 + \gamma_{-e} M_e c^2 + U_{e-e}$

$K_e + Mc^2 + K_{e-} + Mc^2 - \frac{GMM}{r}$

$2K_e + 2Mc^2 - \frac{GM^2}{r}$



b) relate state 1 to 3

$E_3 = E_1 + \cancel{W_{surr}}^{\,0}$

$2mc^2 + 2K_m + \cancel{U_{mm}}^{\,0} = 2Mc^2$

$K_m = (M-m)c^2 = (\gamma-1)mc^2$

3-30-2016

Problem P71 (Cont'd)

$M - \cancel{m} = \gamma m - \cancel{m} \Rightarrow \gamma = \frac{M}{m} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

or $\boxed{v = c\sqrt{1-\left(\frac{m}{M}\right)^2}}$

if $M >> m$, then $\boxed{v \to c}$

c) $v = c\sqrt{1-\left(\frac{m_{pion}}{m_p}\right)^2} = c\sqrt{1-\left(\frac{2.5\times10^{-28}}{6.7\times10^{-27}}\right)^2}$

$= c\sqrt{1-1.39e{-3}} = \boxed{0.9993c} \to \gamma = 26.73$

d) want ratio $\frac{U_e}{K} < 0.1\% = \frac{k_e e e/r}{(\gamma-1)mc^2}$

$r = \frac{k_e ee}{(\gamma-1)mc^2 \, 1000} = \frac{9\times10^9(1.6\times10^{-19})^2}{(26.73-1)(2.5\times10^{-28})(3\times10^8)^2 \, 1000}$

$= \boxed{4\times10^{-19}\,m} \leftarrow$ some small number

Problem P73

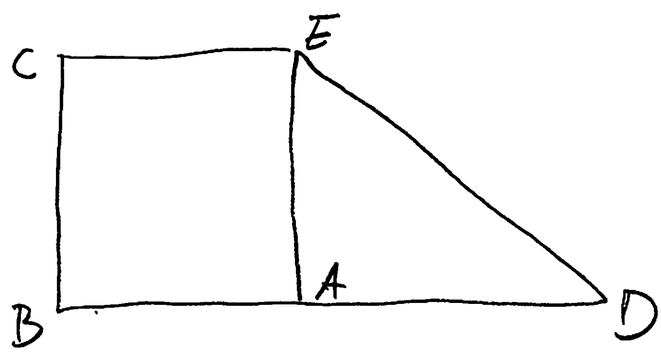
$\vec{r}_A = \langle 0, 0, 0\rangle$

$\vec{r}_B = \langle -3, 0, 0\rangle$

$\vec{r}_C = \langle -3, 13, 0\rangle$

$\vec{r}_D = \langle 33, 0, 0\rangle$

$\vec{r}_E = \langle 0, 13, 0\rangle$



$m = 80$ kg

3-30-2016

Problem P73

a) $\Delta\vec{r}_{AE} = \vec{r}_E - \vec{r}_A = \langle 0, 13, 0\rangle - \langle 0, -0, 0\rangle = \langle 0, 13, 0\rangle$ m

b) $\vec{F}_{AE} = m\langle 0, -g, 0\rangle = \langle 0, -784N, 0\rangle N$

c) $W_{AE} = \vec{F}\cdot\Delta\vec{r}_{AE} = \langle 0, -784, 0\rangle\langle 0, 13, 0\rangle$
$= 0 + (-784)(13) + 0 = \boxed{-10{,}192\ J}$

d) $W_{AB} = \vec{F}\cdot\Delta\vec{r}_{AB} = \boxed{0}$ since $\perp$

e) $W_{BC} = W_{AE} = \boxed{-10{,}192\ J}$

f) $W_{CE} = \boxed{0}$

g) $W_{AE,2} = \boxed{-10{,}192 J}$

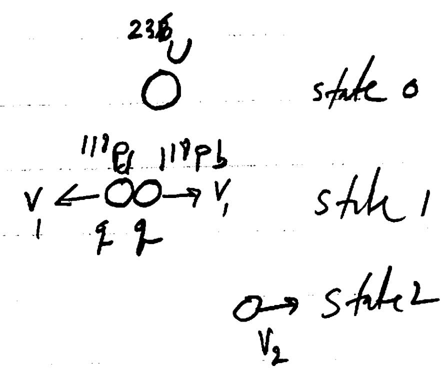
h) $W_{AD} = \boxed{0}$, i) $W_{DE} = \boxed{-10{,}192 J}$, j) $W_{AE} = \boxed{-10{,}192 J}$

k) Work is the same independent of path

Problem P75

$m_U = 235.996u$

$m_{Pd} = 117.994u$



a) $K_f = K_i + \cancel{W_{surr}}^{0}$

$2m_{Pb}c^2 + 2K_{Pb} + U_{Pb-Pb} = m_U c^2$ $V_2 \leftarrow 0$

$2K_{Pb} = [m_U - 2M_{Pb}]c^2 - \frac{k_e q_1 q_2}{r}$

3-30-2016

Problem P75 (cont'd)

For $r \to \infty$

$$K_{Pd} = \frac{1}{2}\left[m_U - 2m_{Pd}\right]c^2 = (\gamma - 1) m_{Pd} c^2$$

or $$\gamma - 1 = \frac{1}{2m_{Pd}}\left[m_U - 2m_{Pd}\right] = \left[\frac{m_U}{2m_{Pd}} - 1\right]$$

$$(\gamma - 1) = \left[\frac{235.996}{2(117.894)} - 1\right] \Rightarrow \gamma = 1.00088, \; v << c$$

$$\frac{1}{2} m_{Pd} V^2 = \left[\frac{m_U}{2} - m_{Pd}\right]c^2$$

$$V = \sqrt{\left[\frac{m_U}{m_{Pd}} - 2\right]}\, c = \boxed{1.26 \times 10^{7}\ m/s}$$

b) $R = 1.3 \times 10^{-15} N^{1/2} = \boxed{6.375 \times 10^{-15}\ m}$ state 1

$N = 118$

$E_0 = E_1$

b) $$m_U c^2 = 2 m_{Pd} c^2 + \frac{k_e (46e)(46e)}{r}$$

$$\frac{\left[m_U - 2m_{Pd}\right]c^2}{k_e 46^2 e^2} = \frac{1}{r}$$

$$r = \frac{k_e 46^2 e^2}{\left[m_U - 2m_{Pd}\right]c^2} = \frac{9 \times 10^9}{\left[235.996 - 2(117.894)\right]}\left[\frac{46 \cdot 1.602 \times 10^{-19}}{3 \times 10^8}\right]^2$$

$$\cdot \frac{1u}{1.7 \times 10^{-27} kg}$$

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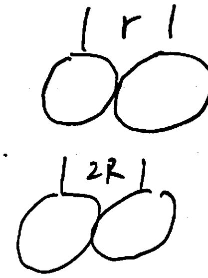
Problem P75 (cont'd)



$1{,}54\times10^{-14}$ part b

$1{,}275\times10^{-14}$ part c

$$r = \boxed{1{,}54\times10^{-14}\,m}$$

$$r-2R \cong \boxed{3\times10^{-15}\,m}$$

d) $2K_{Pd} = [m_U - 2m_{Pd}]c^2$ per Uranium atom

For 1 mole $[m_U - 2m_{Pd}]c^2 A_v$

$$= [235{,}996 - 2(117{,}894)]\left(3\times10^8\,\tfrac{m}{s}\right)^2\left(\frac{1{,}7\times10^{-27}\,kg}{1\,u}\right)$$

$$\times\left(6.02\times10^{23}\ atoms/mol\right)$$

$$= \boxed{1{,}9\times10^{13}\,J}$$