Damped Harmonic Motion

- Simple harmonic motion in which the amplitude is steadily decreased due to the action of some non-conservative force(s), i.e. friction or air resistance (**F**=-b**v,** where b (or C) is the damping coefficient)

- 3 classifications of damped harmonic motion:

1. Underdamped – oscillation, but amplitude decreases with each cycle (shocks)
2. Critically damped – no oscillation, with smallest amount of damping
3. Overdamped – no oscillation, but more damping than needed for critical

- Apply Newton's 2nd Law

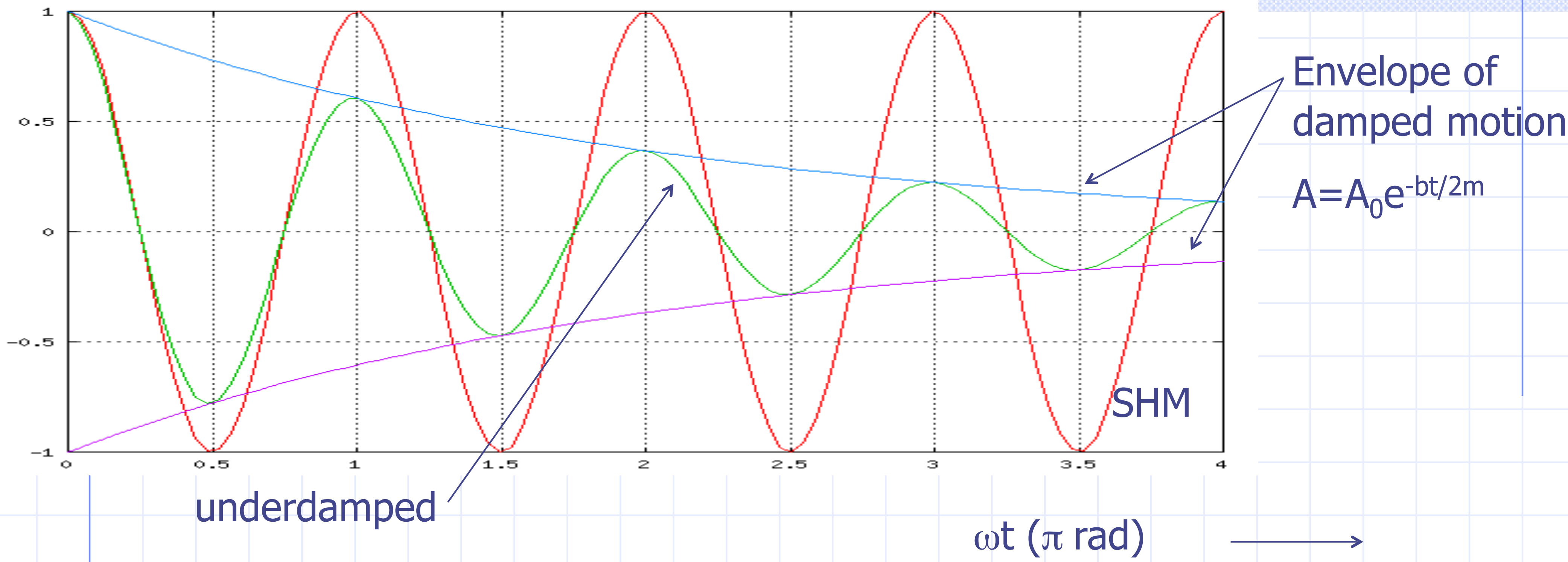
$$\sum F_x = -kx - b\mathrm{v} = ma_x$$

$$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

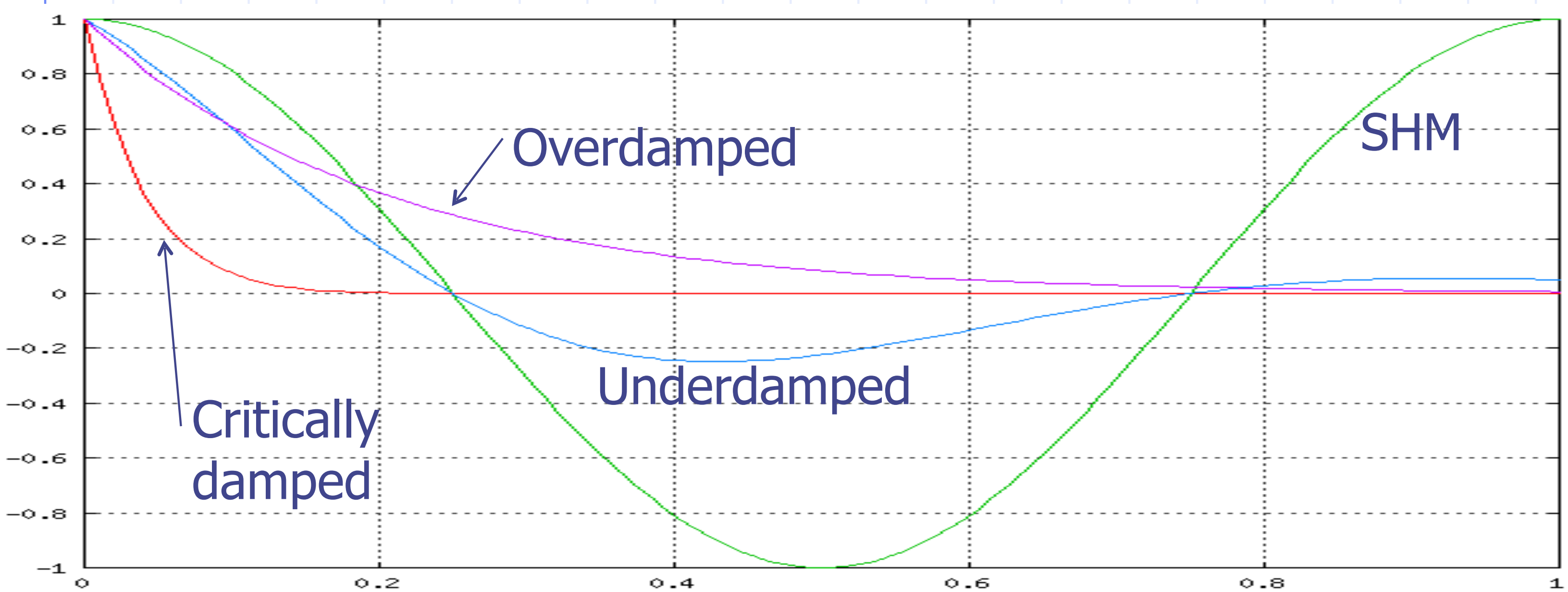
$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

- Another 2nd-order ordinary differential equation, but with a 1st-order term
- The solution is $x = A_0 e^{-\frac{b}{2m}t}\cos(\omega t + \phi_0)$
- Where $\omega = \sqrt{\omega_0^2 - (b/2m)^2}$ $\qquad \omega_0 = \sqrt{k/m}$
- Type of damping determined by comparing

ω_0 and $b/2m$



Envelope of damped motion
A=A0e-bt/2m
SHM
underdamped
ωt (π rad)

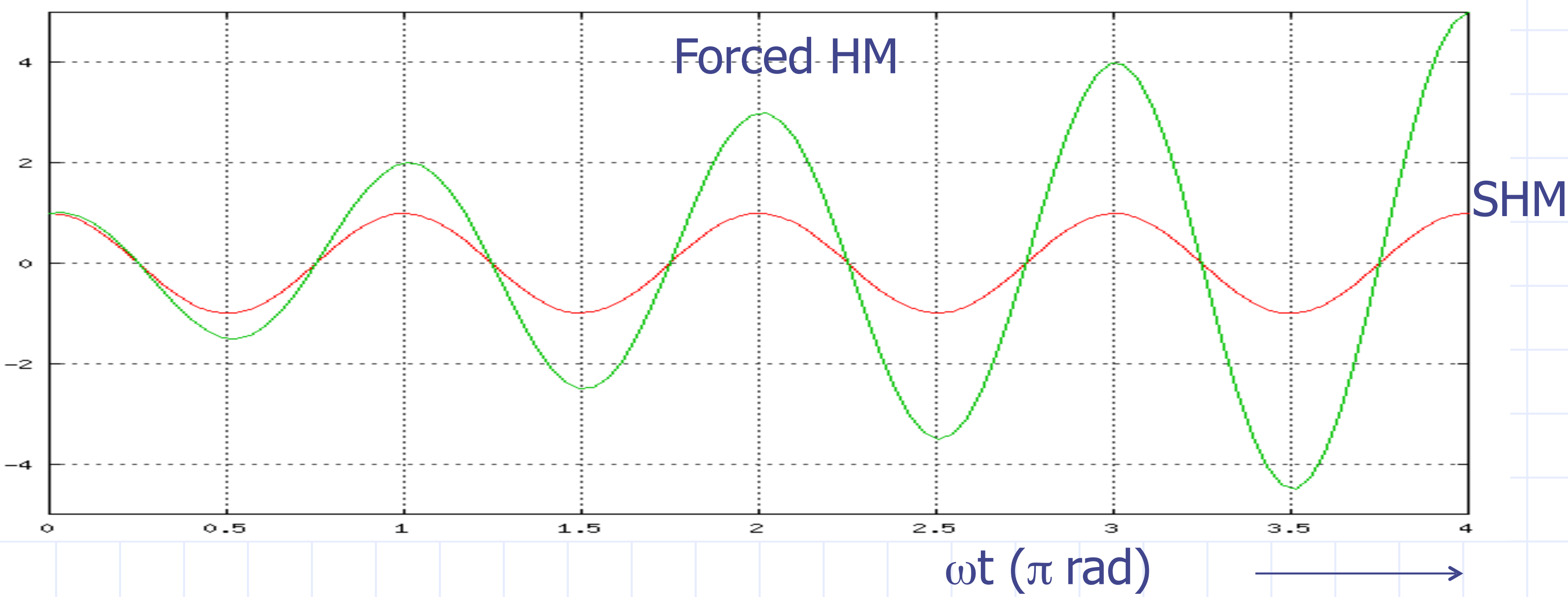


Overdamped
SHM
Underdamped
Critically damped

Forced Harmonic Motion

- Unlike damped harmonic motion, the amplitude may increase with time

- Consider a swing (or a pendulum) and apply a force that increases with time; the amplitude will increase with time



- Consider the spring-mass system, it has a frequency

$$f = \frac{1}{2\pi}\sqrt{k/m} = f_0$$

- We call this the natural frequency f_0 of the system. All systems (car, bridge, pendulum, etc.) have an f_0

- We can apply a time-dependent external driving force with frequency f_d ($f_d \neq f_0$) to the spring-mass system

$$F(t) = F_{\mathbf{ext}} \cos(2\pi\ f_d t)$$

- This is forced harmonic motion, the amplitude increases

- But if $f_d = f_0$, the amplitude can increase dramatically – this is a condition called <u>resonance</u>

- Examples: a) out-of-balance tire shakes violently at certain speeds,

  b) Tacoma-Narrows bridge's f_0 matches frequency of wind