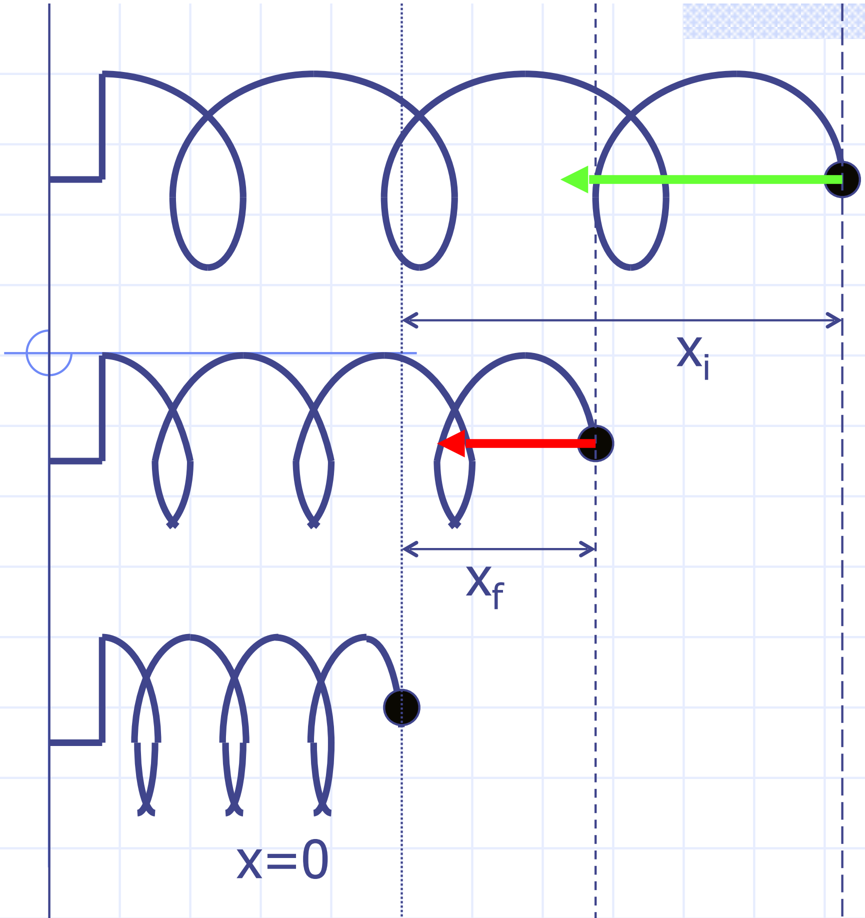
Chapter 7 Internal Energy

Work done by a spring

- We know that work equals force times displacement

$$F_s = -kx$$

Hooke's Law for the restoring force of an ideal spring. (It is a conservative force.)



$$s = x_f - x_i$$

But the force is not constant

$$F_{s,i} = -kx_i,$$
$$F_{s,f} = -kx_f$$

Take the average force

$$F_{s,avg} = \frac{F_{s,i} + F_{s,f}}{2}$$

$$F_{s,avg} = -\tfrac{1}{2}k(x_f + x_i)$$

Then the work done by the spring is

$$W_s = F_{s,avg}\cos\phi\, s = F_{s,avg}\cos 0^\circ\ s$$

$$= -\tfrac{1}{2}k(x_f + x_i)(x_f - x_i) = -\tfrac{1}{2}k(x_f^2 - x_i^2)$$

$$\boxed{W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2}$$

$= -\Delta U$

$$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 = U_{s,i} - U_{s,f}$$

$$U_{elastic} = U_s = \tfrac{1}{2}kx^2$$

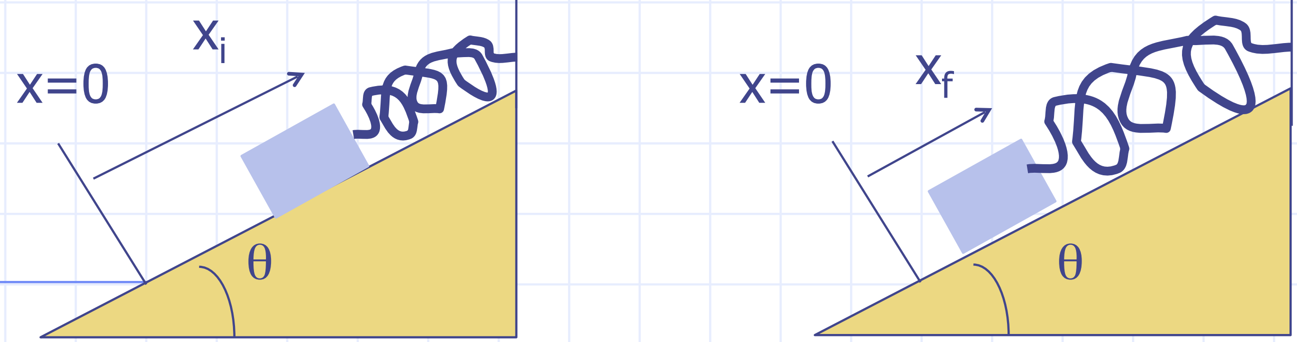
Units of N/m m² = N m = J

- Total potential energy is

$$U_{total} = U_g + U_s = mgy + \tfrac{1}{2}kx^2$$

Example Problem

A block (m = 1.7 kg) and a spring (k = 310 N/m) are on a frictionless incline (θ = 30°). The spring is compressed by x_i = 0.31 m relative to its unstretched position at x = 0 and then released. What is the speed of the block when the spring is still compressed by x_f = 0.14 m?



Given: m=1.7 kg, k=310 N/m, θ=30°, x_i=0.31 m, x_f=0.14 m, frictionless

Method: no friction, so we can use conservation of energy

$$E = \tfrac{1}{2} m v^2 + mgh + \tfrac{1}{2} k x^2$$

Initially

$$v_i = 0, h_i = x_i \sin\theta$$

$$E_i = m g x_i \sin\theta + \tfrac{1}{2} k x_i^2$$

Finally $h_f = x_f \sin\theta$, find v_f

$$E_f = \tfrac{1}{2} m v_f^2 + mgx_f \sin\theta + \tfrac{1}{2} k x_f^2$$

$$E_f = E_i$$

$$\tfrac{1}{2} m v_f^2 + mgx_f \sin\theta + \tfrac{1}{2} k x_f^2$$

$$= mgx_i \sin\theta + \tfrac{1}{2} k x_i^2$$

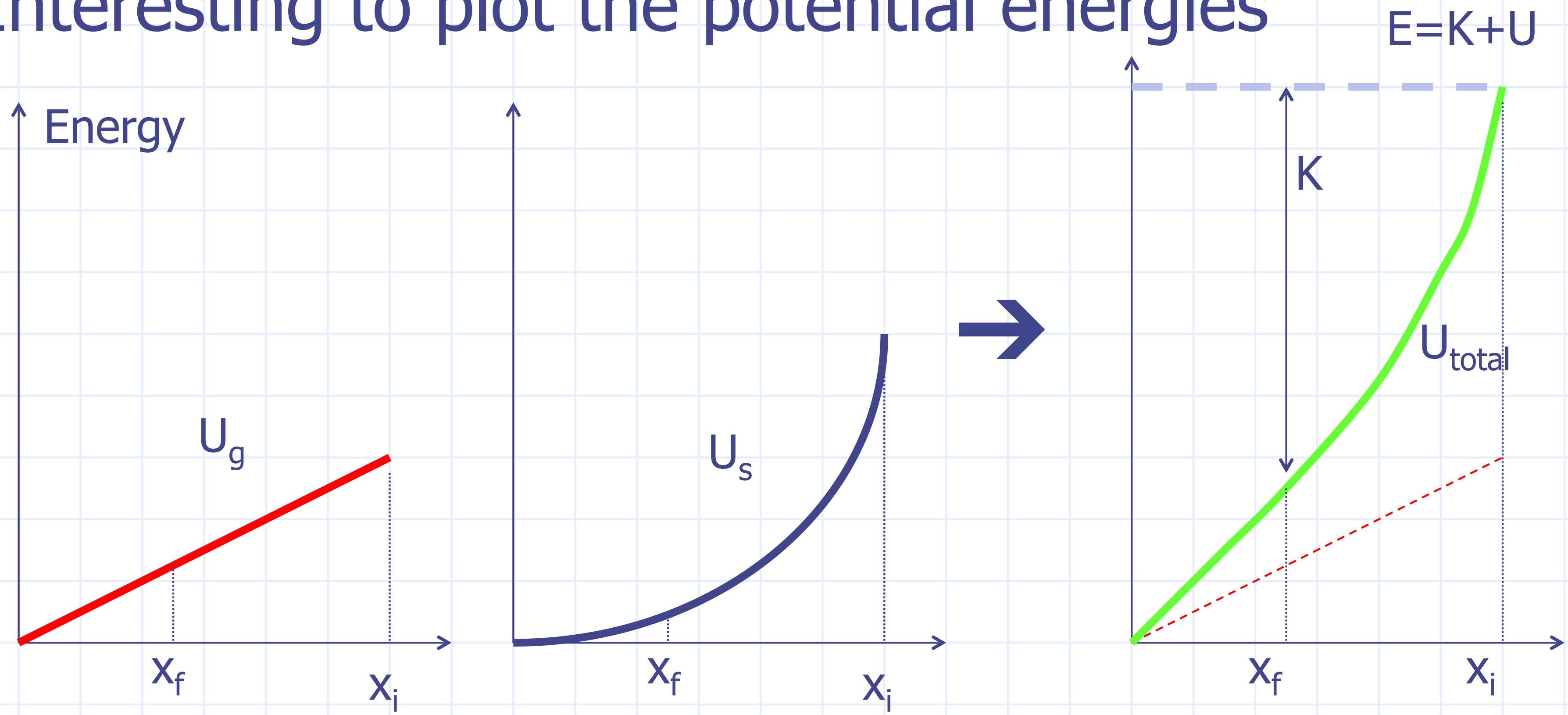
$$\tfrac{1}{2} m v_f^2 = mg(x_i - x_f)\sin\theta + \tfrac{1}{2} k (x_i^2 - x_f^2)$$

$$v_f = \sqrt{2g(x_i - x_f)\sin\theta + \frac{k}{m}(x_i^2 - x_f^2)}$$

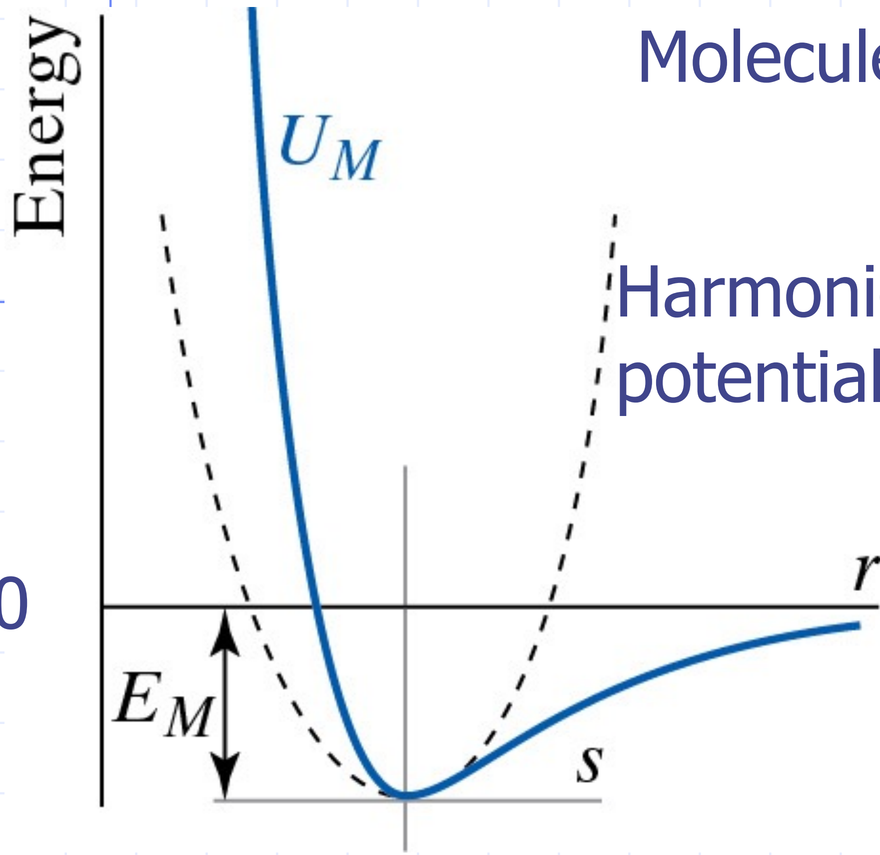
$$v_f = \sqrt{2(9.8)(0.31-0.14)\sin 30^\circ + \frac{310}{1.7}(0.31^2 - 0.14^2)}$$

$$v_f = \sqrt{1.666 + 13.95} = \boxed{3.95\,\frac{m}{s}}$$

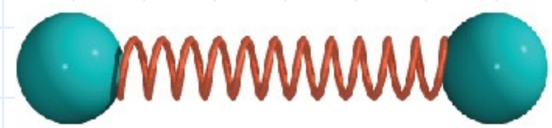
Interesting to plot the potential energies



Molecular potentials (two atoms)



Molecule AB



Harmonic oscillator potential

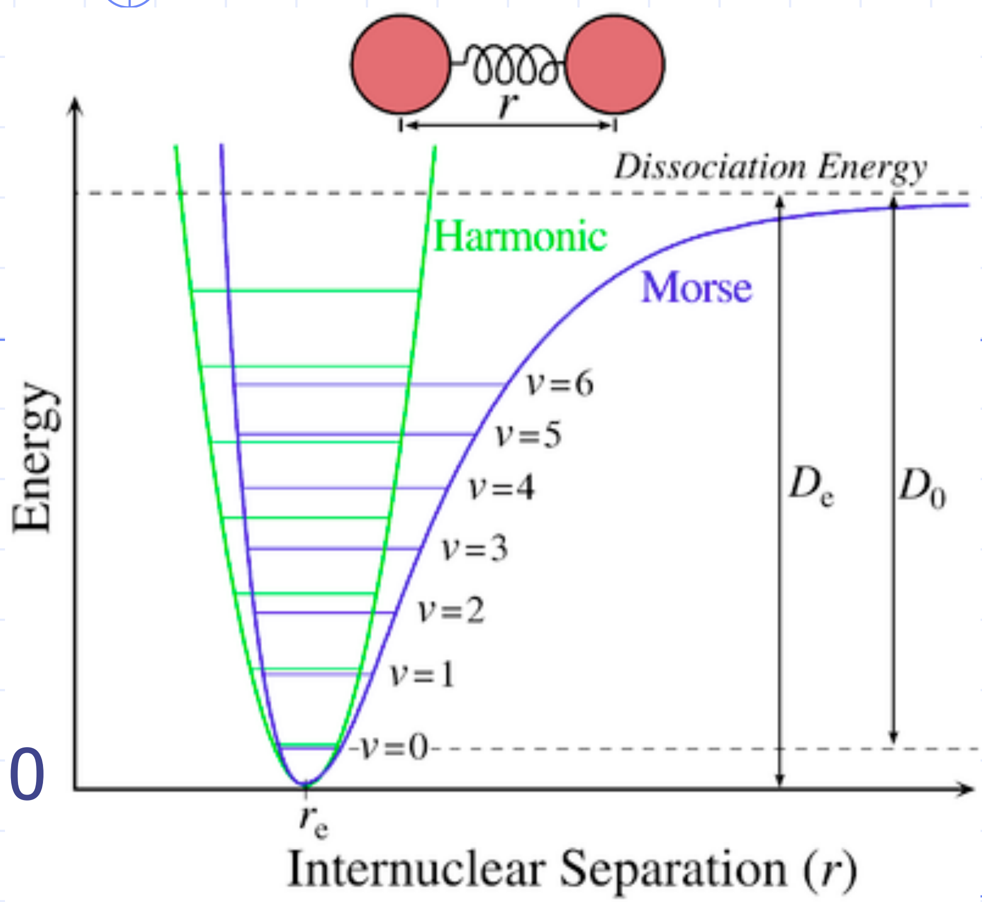
A + B

0

r_e

$$U_{\mathrm{M}} = \frac{1}{2}k_s(r - r_e)^2 - E_{\mathrm{M}}$$

Morse potential



Better approximation, but need EM, re, α, where

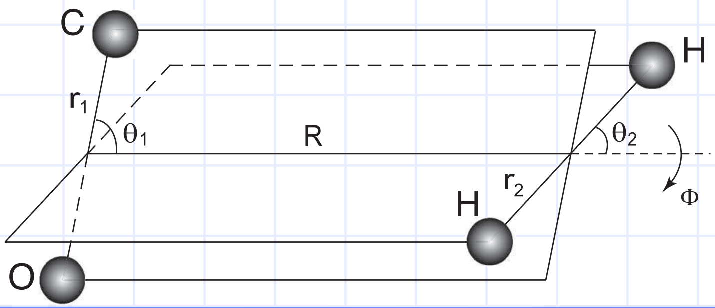
$$\alpha = \sqrt{k_s/2E_{\mathrm{M}}}$$

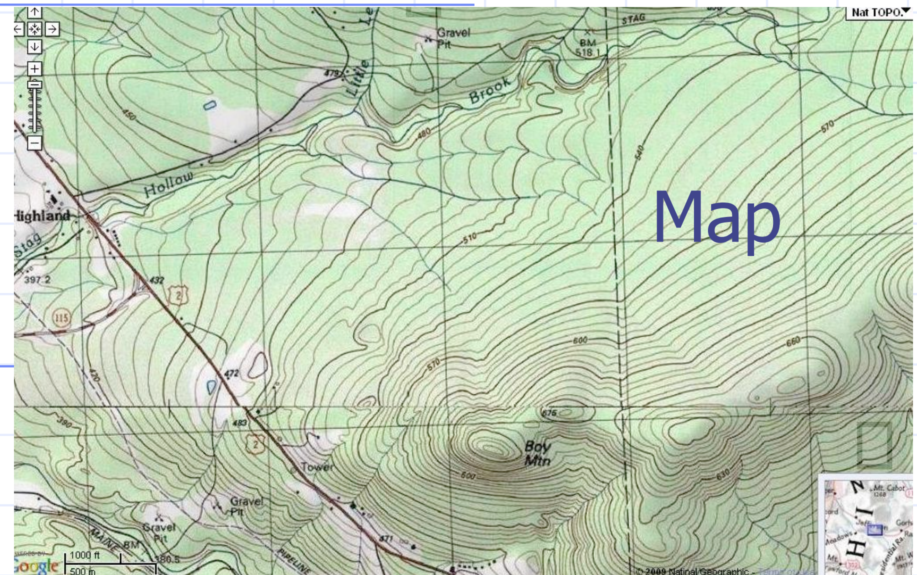
But at large r, Morse potential does poorly

$$U_{\mathrm{M}} \rightarrow \frac{C_6}{r^6}$$

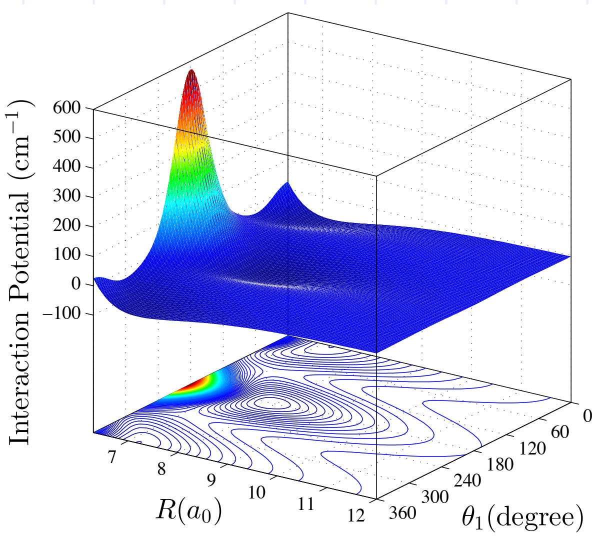
$$U_{\mathrm{M}}^{\mathrm{Morse}} = E_{\mathrm{M}}[1 - \exp[-\alpha(r - r_e)]]^2$$

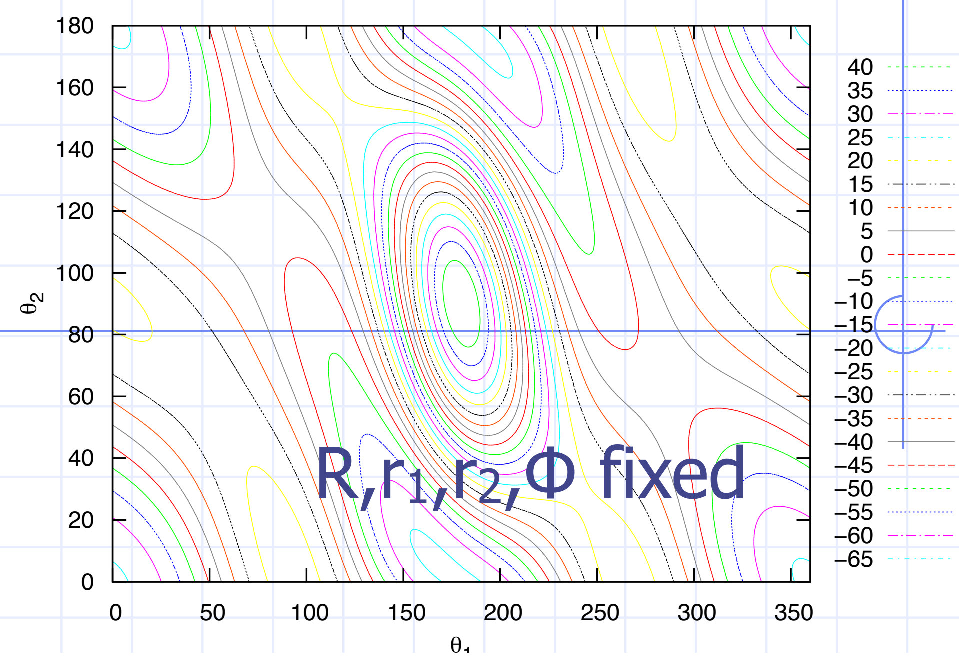
Multidimensional potentials





H_2-CO, $U(R, r_1, r_2, \theta_1, \theta_2, \Phi)$





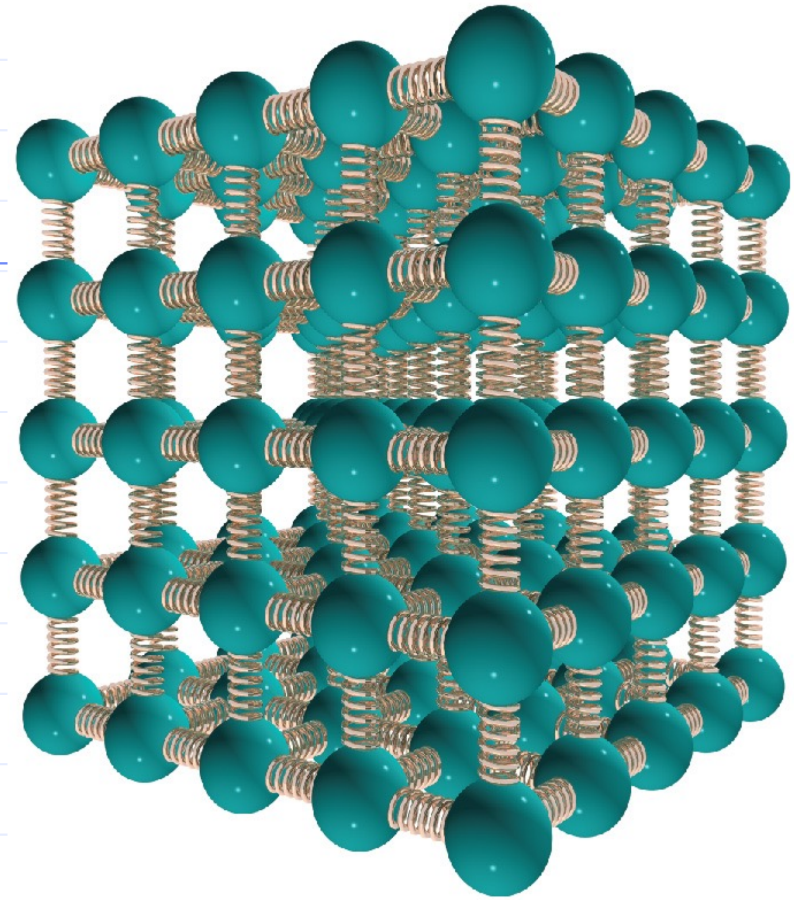
r_1, r_2, θ_2, Φ fixed

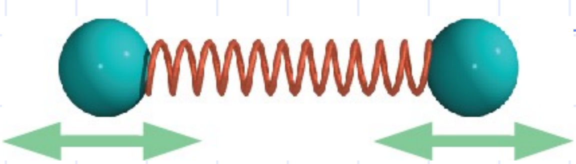
Example problem

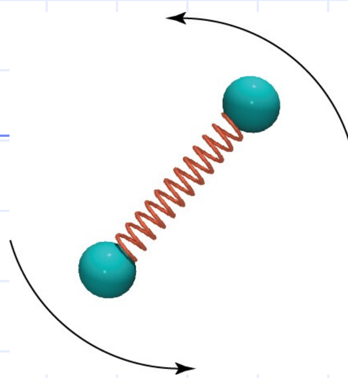
- If it takes 4.00 J of work to stretch a Hooke's law spring 10.0 cm from its unstretched length, determine the extra work required to stretch it an additional 10.0 cm.

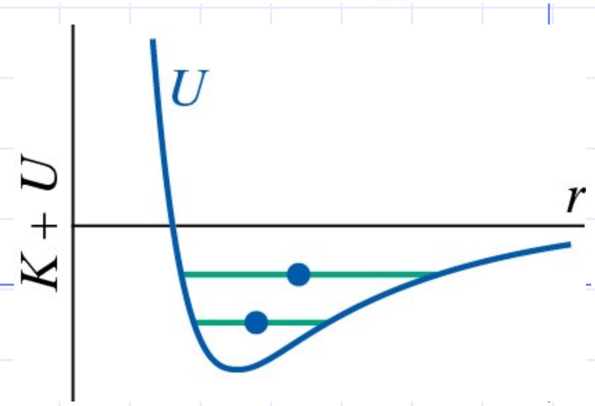
Thermal energy

- Modeling a solid as a collection of spring-masses
- Internal energy due to K of atoms and U of "springs"









Conservative and Non-conservative Forces

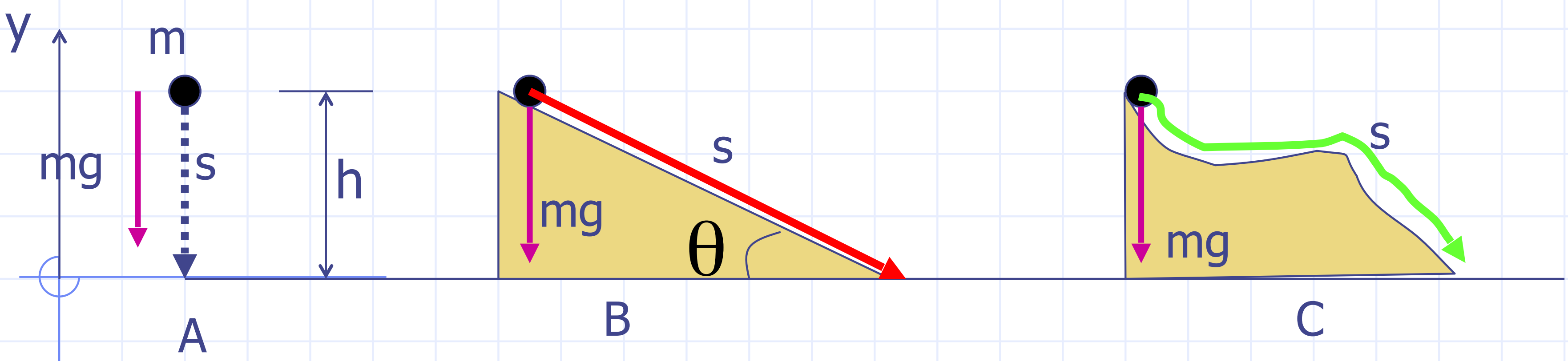
- Conservative Force: a force for which the work it does on an object does not depend on the path. Gravity is an example.

- We know we can obtain the work with the work integral.

$$W = \int_{x_i}^{x_f} F_x dx = W_c$$

- If the force is conservative, then $W=W_c$ and this work can be related to the change in potential energy

$$W_c = \int_{x_i}^{x_f} F_x dx = -\Delta U$$

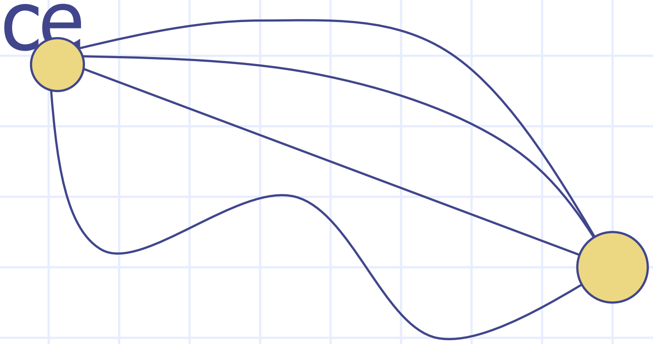


A $$W = F\cos\phi \, s = mgh$$

B $$W = (mg\sin\theta)s = (mg\sin\theta)\frac{h}{\sin\theta} = mgh$$

C $$W = mgh$$

- Non-conservative Force - a force for which the work done depends on the path
  - friction
  - air resistance



- If the force is conservative, we can find the potential energy due to the force

$$U_f(x) = -\int_{x_i}^{x_f} F_x dx + U_i(x)$$

it is usually convenient to take $U_i(x)=0$

- Or if we know U(x) and the force is conservative, we can obtain F

$$dU(x) = -F_x dx \Rightarrow F_x = -\frac{dU(x)}{dx}$$

- The x-component of a conservative force equals the negative derivative of the potential energy with respect to x

If both conservative and non-conservative forces act on an object, the work-energy theorem is modified

$$W_{total} = W_C + W_{NC} = K_f - K_i$$

$$W_{NC} = K_f - K_i - W_C$$

For the case of gravity

$$W_g = W_C = mg(y_i - y_f)$$

$$W_{NC} = K_f - K_i - mg(y_i - y_f)$$

$$W_{NC} = K_f - K_i - mgy_i + mgy_f$$

$$\boxed{W_{NC} = \Delta K + \Delta U}$$

$$= K_f - K_i + U_f - U_i$$

$$= (K_f + U_f) - (K_i + U_i)$$

$$W_{NC} = E_f - E_i = W_{surr}$$

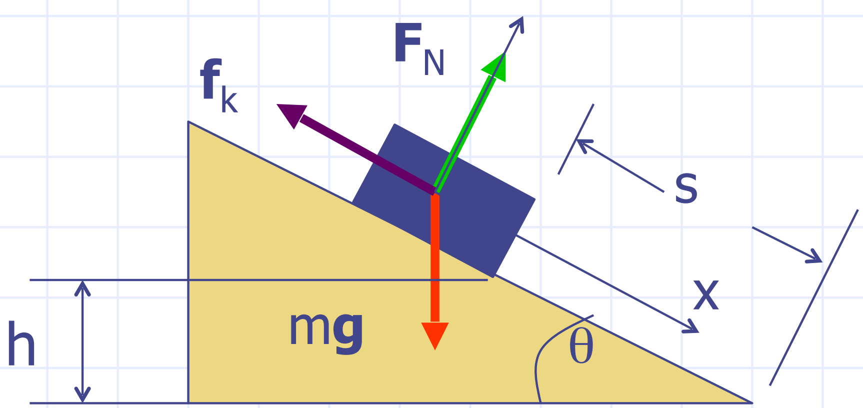
- If no net non-conservative forces

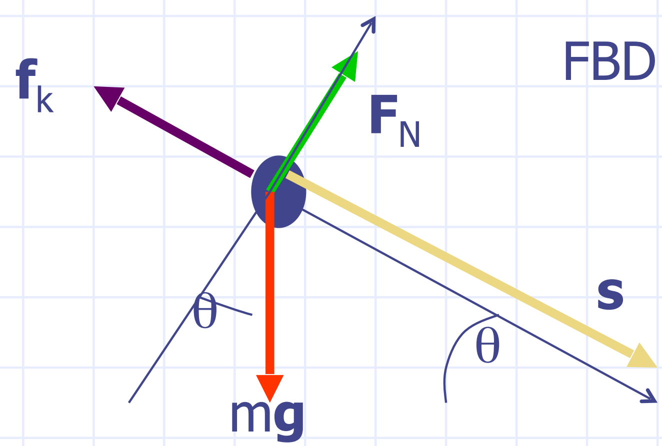
$$W_{NC} = 0 \Rightarrow E_f = E_i = E$$

- Then, conservation of mechanical energy holds

$$\Delta K = -\Delta U$$

Crate on Incline Revisited





$$W_N = 0$$
$$W_g = mg\sin\theta\ s = mgh = W_C$$
$$W_f = -\mu_k mg\cos\theta\ s = W_{NC}$$

- The crate starts from rest, $v_i=0$

$$K_i = 0, E_i = U_i = mgh = W_g$$
$$W_{NC} = E_f - E_i$$
$$E_f = E_i + W_{NC} = mgh - \mu_K mg\cos\theta\ s$$
$$\boxed{E_f < E_i}$$

- Some energy, W_{NC} is loss from the system

- In this case it is due to the non-conservative friction force → energy loss in the form of heat

- Because of friction, the final speed is only 9.3 m/s as we found earlier
- If the incline is frictionless, the final speed would be:

$$E_f = E_i, \text{since } K_i = 0, U_f = 0$$

$$\tfrac{1}{2} m v_f^2 = mgh = W_g$$

$$v_f = \sqrt{\frac{2W_g}{m}} = \sqrt{\frac{2(7510\text{ J})}{100\text{ kg}}} = \boxed{12.3\,\tfrac{\text{m}}{\text{s}}}$$

- Because of the loss of energy, due to friction, the final velocity is reduced. It seems that energy is not conserved

Conservation of Energy

- There is an overall principle of conservation of energy
- Unlike the principle of conservation of mechanical energy, which can be “broken”, this principle **can not**
- It says: ``The total energy of the Universe is, has always been, and always will be constant. Energy can neither be created nor destroyed, only converted from one form to another.”
- So far, we have only been concerned with mechanical energy

- There are other forms of energy: heat, electromagnetic, chemical, nuclear, rest mass ($E_m = mc^2$)

$$E_i^{total} = E_f^{total}$$

$$E_i^{mech} + E_i^{others} = E_f^{mech} + E_f^{others}$$

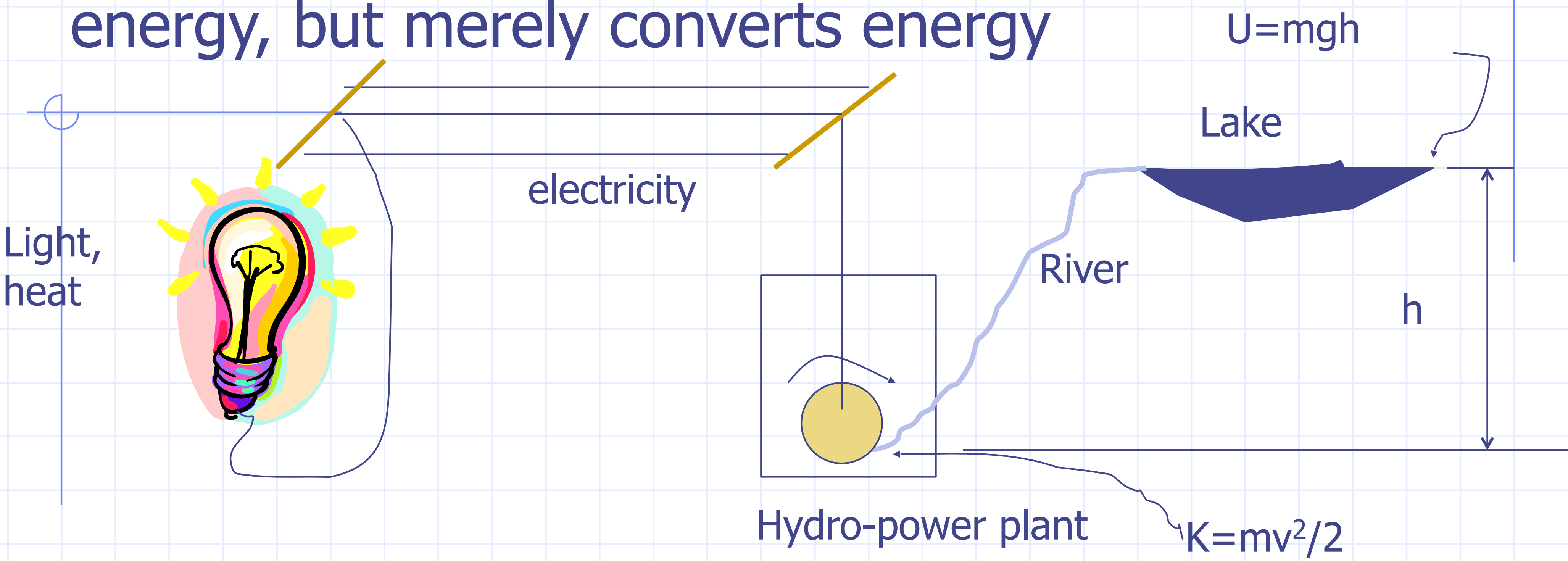
$$E_f^{mech} = E_i^{mech} + E_i^{others} - E_f^{others}$$

$$E_f^{mech} = E_i^{mech} + W_{NC}$$

$$\Rightarrow W_{NC} = E_i^{others} - E_f^{others} = Q$$

- Q (W_{NC}) is the energy lost (or gained) by the mechanical system

- The electrical utility industry does not produce energy, but merely converts energy



Example Problem

A ball is dropped from rest at the top of a 6.10-m tall building, falls straight downward, collides inelastically with the ground, and bounces back. The ball loses 10.0% of its kinetic energy every time it collides with the ground. How many bounces can the ball make and still reach a window sill that is 2.44 m above the ground?

Solution:

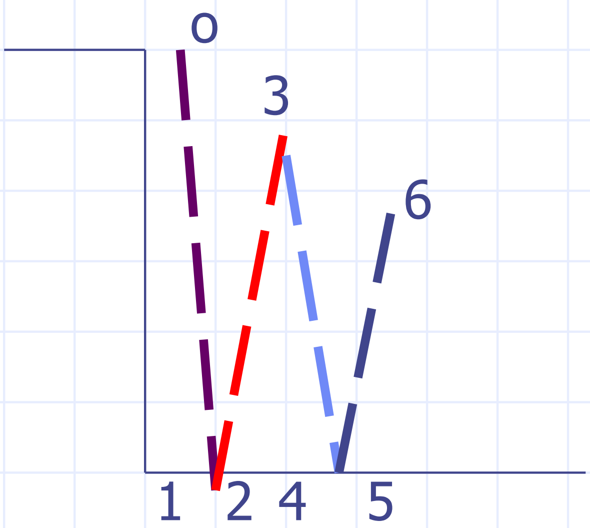
Method: since the ball bounces on the ground, there is an external force. Therefore, we can **not** use conservation of linear momentum.

An inelastic collision means the total energy is not conserved, but we know by how much it is not conserved. On every bounce 10% of K is lost:

$Q_i = 0.1K_i, i = 1,2,3,\ldots,n$ $\quad n$ **is the number of bounces**

Given: h_0 = 6.10 m, h_f = 2.44 m

$$E_o = U_o = mgh_o$$
$$E_1 = K_1 = E_o$$



Since energy is conserved from point 0 to point 1.

However, between point 1 and 2, energy is lost

$$Q_2 = 0.1K_1$$
$$W_{NC} = -Q = E_{final} - E_{initial}$$
$$-Q_2 = E_2 - E_1$$
$$-0.1K_1 = E_2 - E_1$$
$$-0.1E_o = E_2 - E_o$$
$$E_2 = 0.9E_o = E_3 = E_4$$

Total energy after one bounce

By the same reasoning

$$E_5 = 0.9E_4 = 0.9E_2 = 0.9(0.9E_o)$$

$$E_5 = 0.9^2 E_o$$ Total energy after two bounces

The total energy after n bounces is then

$$E_f = 0.9^n E_o$$

$$mgh_f = 0.9^n mgh_o$$

$$h_f = 0.9^n h_o$$

$$0.9^n = h_f / h_o$$

$$\log(0.9)^n = \log(h_f / h_o)$$

$$n \log(0.9) = \log(h_f / h_o)$$

$$n = \frac{\log(h_f / h_o)}{\log(0.9)}$$

$$n = \frac{\log(2.44 / 6.10)}{\log(0.9)}$$

$$n = 8.7$$

Answer is 8 bounces

Power

Average power:

$$P_{avg} = \frac{W}{\Delta t}$$

Units of J/s=Watt (W)

Measures the rate at which work is done

or

$$P_{avg} = \frac{F\cos\phi\, s}{\Delta t} = F\cos\phi\, \mathrm{v}_{\mathrm{avg}}$$

Instantaneous power:

$$P = \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt}$$

$$P = \frac{\vec{F}\cdot d\vec{s}}{dt} = \vec{F}\cdot\vec{\mathrm{v}}$$

W can also be replaced by the total energy E. So that power would correspond to the rate of energy transfer

Example

A car accelerates uniformly from rest to 27 m/s in 7.0 s along a level stretch of road. Ignoring friction, determine the average power required to accelerate the car if (a) the weight of the car is 1.2×10^4 N, and (b) the weight of the car is 1.6×10^4 N.

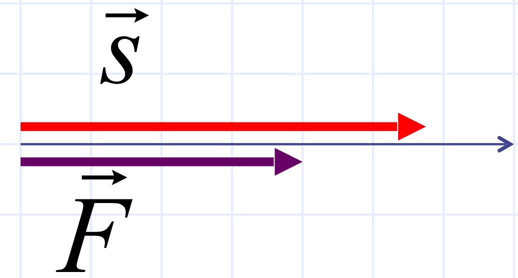
Solution:

Given: $v_i = 0$, $v_f = 27$ m/s, $\Delta t = 7.0$ s,

(a) $mg = 1.2 \times 10^4$ N, (b) $mg = 1.6 \times 10^4$ N

Method: determine the acceleration

$$P_{avg} = \frac{W}{\Delta t} = \frac{F\cos\phi\, s}{\Delta t}$$



- We don't know the displacement **s**
- The car's motor provides the force **F** to accelerate the car – **F** and **s** point in same direction

Need a_s and s

$$P_{avg} = \frac{Fs}{\Delta t} = \frac{ma_s s}{\Delta t}$$

$$v_f^2 = v_i^2 + 2a_s s \Rightarrow a_s s = \frac{(v_f^2 - v_i^2)}{2}$$

$$P_{avg} = \frac{m}{\Delta t}\left(\frac{(v_f^2 - v_i^2)}{2}\right) = \frac{m}{(7.0\ s)}\left(\frac{27^2 - 0}{2}\right) = 52m$$

$$P_{avg} = \frac{52mg}{g} = \frac{52(1.2x10^4)}{9.80} = \boxed{6.4x10^4 \text{ W}}$$ (a)

$$P_{avg} = \frac{52(1.6x10^4)}{9.80} = \boxed{8.5x10^4 \text{ W}}$$ (b)

Or from work-energy theorem

$$W = \Delta K = \tfrac{1}{2} m v_f^2$$

$$P_{avg} = \frac{W}{\Delta t} = \frac{m v_f^2}{2\Delta t}$$ Same as on previous slide