Chapter 5: Forces and Motion

- Uniform circular motion: An object moving on a <u>circular path of radius r</u> at a constant speed v
- As motion is not on a straight line, the direction of the velocity vector is not constant
- The motion is circular

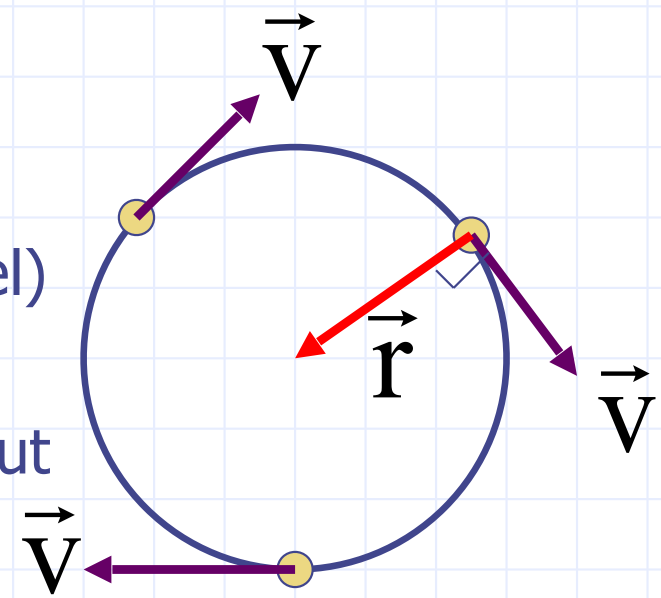
- Compare to:

1D – straight line

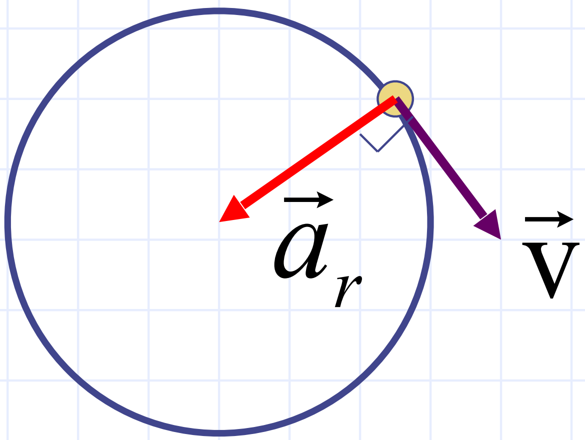
2D – parabola

- Velocity vector is always tangent (parallel) to the circle

- Velocity direction constantly changing, but magnitude remains constant

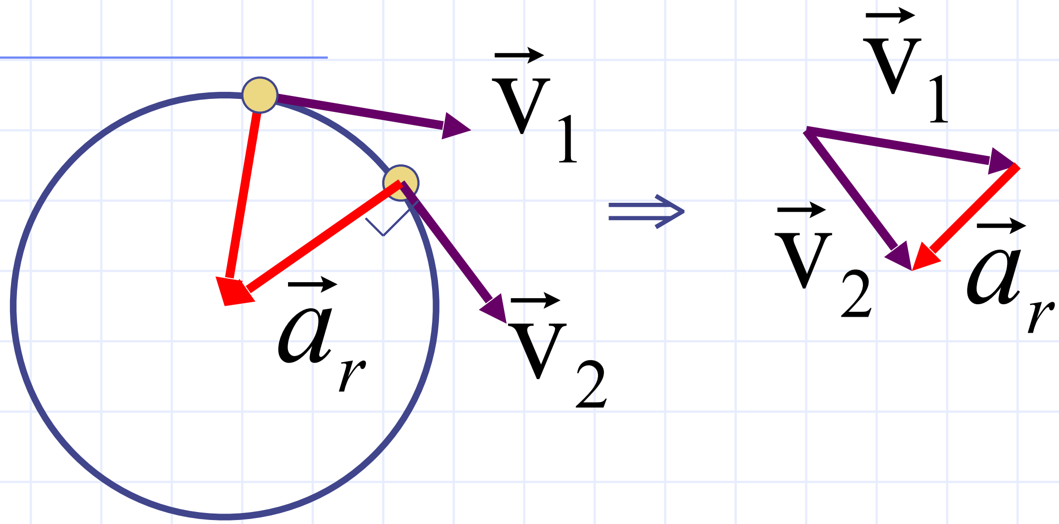


- Vectors **r** and **v** are always perpendicular
- Since the velocity direction always changes, this means that the velocity is not constant (though speed is constant), therefore the object is accelerating
- The acceleration $\mathbf{a}_r$ points radially inward. Like velocity, its direction changes, therefore the acceleration is not constant (though its magnitude is)



- Vectors $\mathbf{a}_r$ and **v** are also perpendicular
- The speed does not change, since $\mathbf{a}_r$ acceleration has no component along the velocity direction

- Why is the acceleration direction radially inward?



Since

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

- This radial acceleration is called the centripetal acceleration

$$a_r = \frac{v^2}{r}$$

- Time to complete a full orbit

$$D = 2\pi r = \text{circumference}$$

$$T = \frac{\text{circumference}}{\text{speed}} = \frac{2\pi r}{\text{v}} = \text{Period}$$

- The Period T is the time (in seconds) for the object to make one complete orbit or cycle

- Find some useful relations for v and a_r in terms of T

$$\boxed{\text{v} = \frac{2\pi r}{T}}$$

$$a_r = \frac{\text{v}^2}{r} = \left(\frac{2\pi r}{T}\right)^2 \frac{1}{r} = \boxed{\frac{4\pi^2 r}{T^2} = a_r}$$

Forces and Circular Motion

- We now look at applying Newton's 2nd law to circular (or curvilinear) motion in a plane
- We introduced the radial (or centripetal) acceleration for uniform circular motion

$$a_r = \frac{v^2}{r}$$

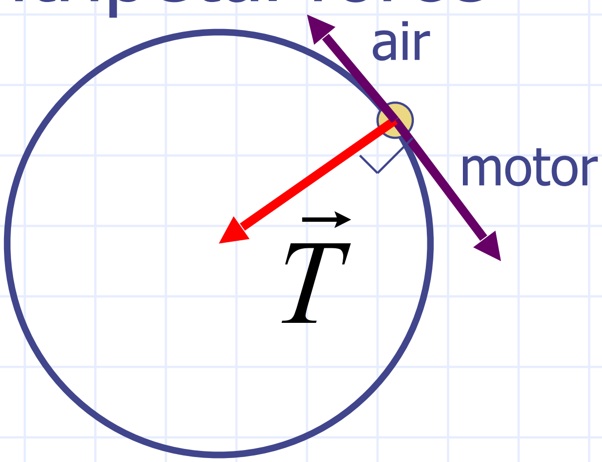
- This acceleration implies a ``force"

$$\sum F_r = ma_r = \frac{mv^2}{r}$$

``centripetal force" (is not a force)

- The ``centripetal force'' is the net force required to keep an object moving on a circular path

- Consider a motorized model airplane on a wire which flies in a horizontal circle, if we neglect gravity, there are only three forces, the force provided by the airplane motor which tends to cause the plane to travel in a straight line, air resistance, and the tension force in the wire, which causes the plane to travel in a circle – the tension is the ``centripetal force''

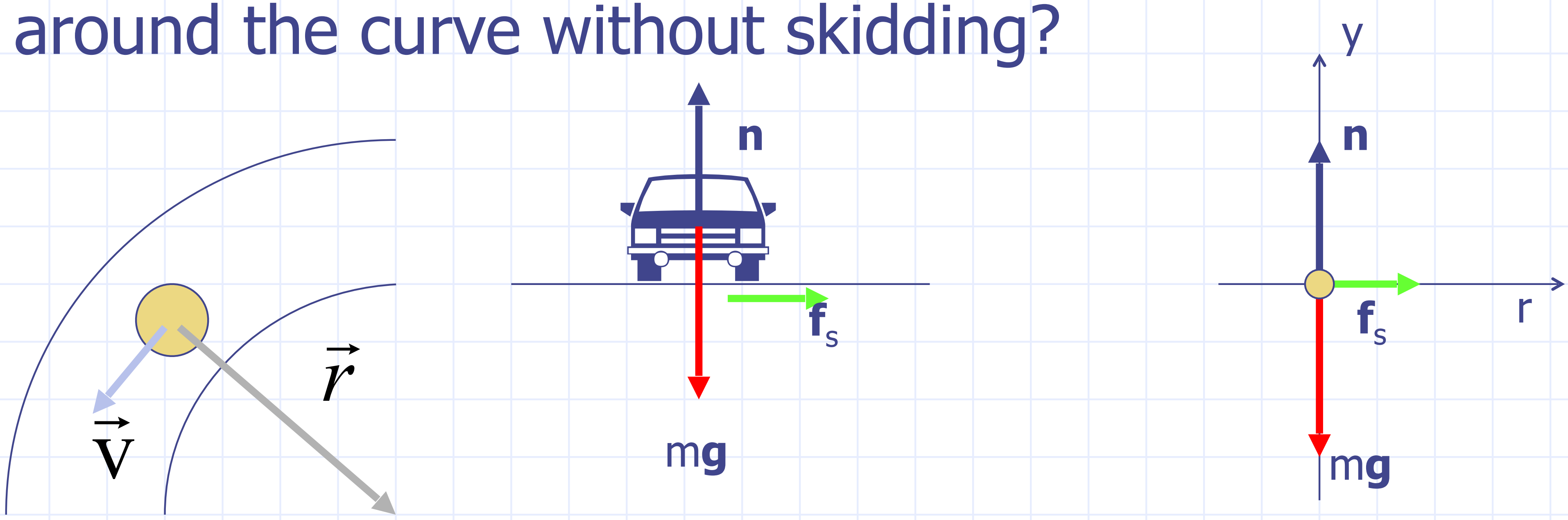


Consider forces in radial direction (positive to center)

$$\sum F_r = ma_r \Rightarrow T = \frac{m\text{v}^2}{r}$$

Example (Simple)

A car travels around a curve which has a radius of 316 m. The curve is flat, not banked, and the coefficient of static friction between the tires and the road is 0.780. At what speed can the car travel around the curve without skidding?



$$\sum F_y = ma_y \qquad \sum F_r = ma_r$$

$$n - mg = 0$$

$$\boxed{n = mg} \qquad \boxed{f_s = \frac{m\text{v}^2}{r}}$$

- Now, the car will not skid as long as f_s is less than the maximum static frictional force

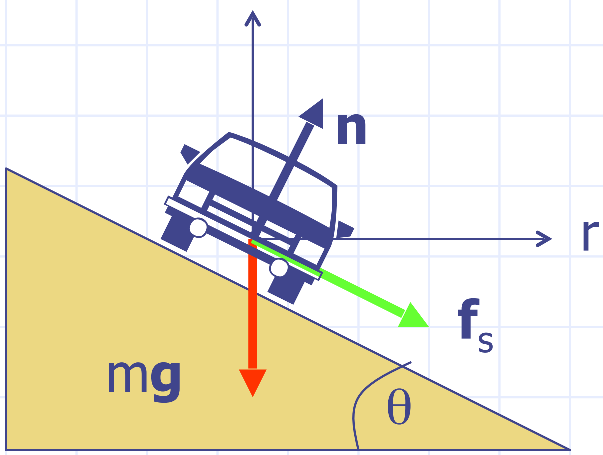
$$\frac{m\text{v}^2}{r} < f_s^{\max} = \mu_s n \Rightarrow \frac{m\text{v}^2}{r} < \mu_s mg$$

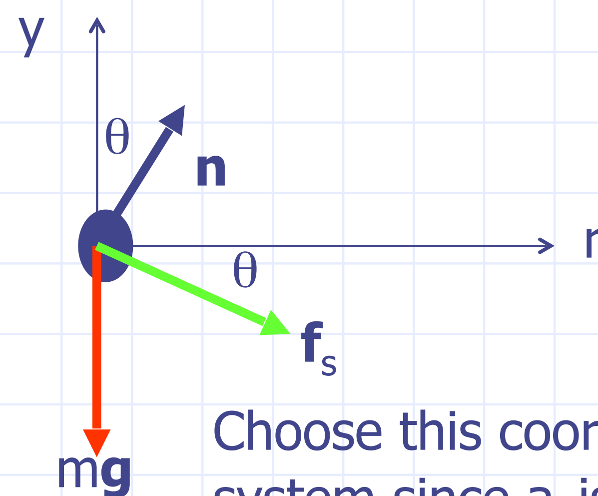
$$\text{v} = \sqrt{\mu_s g r} = \sqrt{(0.780)(9.80\,\tfrac{\text{m}}{\text{s}^2})(316\text{ m})} = 49.1\,\tfrac{\text{m}}{\text{s}}$$

$$= 49.1\,\tfrac{\text{m}}{\text{s}}\left(\frac{3600\text{ s}}{1\text{ hr}}\right)\left(\frac{1\text{ mi}}{1609\text{ m}}\right) = \boxed{110\ \tfrac{\text{mi}}{\text{hr}}}$$

Example (tricky)

- To reduce skidding, use a banked curve. Consider same conditions as previous example, but for a curve banked at the angle θ





Choose this coordinate system since a_r is radial

$$\sum F_y = ma_y$$
$$n\cos\theta - f_s\sin\theta - mg = 0$$

Since acceleration is radial only

- Since we want to know at what velocity the car will skid, this corresponds to the centripetal force being equal to the maximum static frictional force

$$f_s \Rightarrow f_s^{\max} = \mu_s n$$

Substitute into previous equation

$$n\cos\theta - \mu_s n\sin\theta = mg$$

$$n(\cos\theta - \mu_s \sin\theta) = mg \Rightarrow \boxed{n = \frac{mg}{\cos\theta - \mu_s \sin\theta}}$$

$$\sum F_r = ma_r$$

$$n\sin\theta + f_s\cos\theta = \frac{m\mathrm{v}^2}{r}$$

$$\boxed{n(\sin\theta + \mu_s\cos\theta) = \frac{m\mathrm{v}^2}{r}}$$

Substitute for n and solve for v

$$\left(\frac{mg}{\cos\theta - \mu_s \sin\theta}\right)(\sin\theta + \mu_s \cos\theta) = \frac{m\mathrm{v}^2}{r}$$

$$\boxed{\mathrm{v} = \sqrt{rg\frac{(\sin\theta + \mu_s \cos\theta)}{(\cos\theta - \mu_s \sin\theta)}}}$$

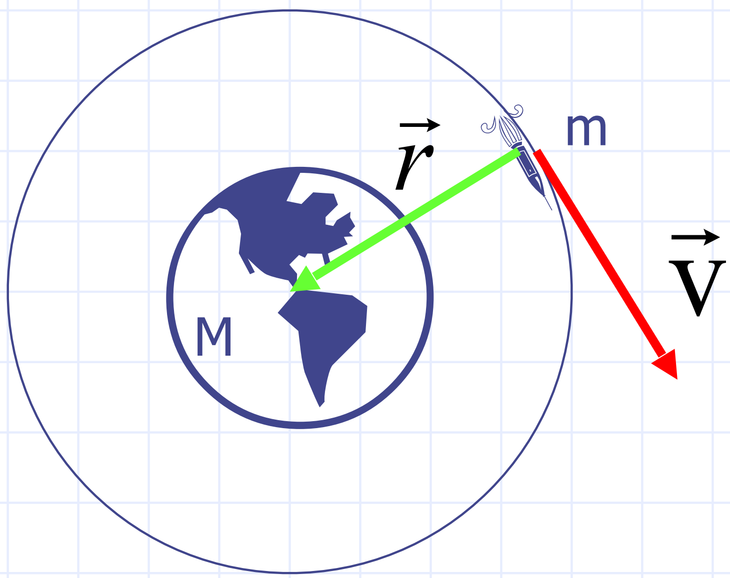
- Adopt r = 316 m and θ = 31°, and μ_s=0.780 from earlier

$$\mathrm{v} = 89.7\ \tfrac{\mathrm{m}}{\mathrm{s}} = \boxed{200\ \tfrac{\mathrm{mi}}{\mathrm{hr}}}$$

- Compare to when μ_s=0

$$\mathrm{v} = \sqrt{rg\frac{(\sin\theta)}{(\cos\theta)}} = \sqrt{rg\tan\theta} = 43.1\ \tfrac{\mathrm{m}}{\mathrm{s}} = \boxed{96.5\ \tfrac{\mathrm{mi}}{\mathrm{hr}}}$$

Orbital Motion of Satellites



- Satellites move in circular (or more generally, elliptical) orbits
- Compute their period and speed by applying Newton's 2nd Law in the radial direction

$$\sum F_r = \frac{m\mathrm{v}^2}{r}$$

Orbital speed

$$\frac{GMm}{r^2} = \frac{m\mathrm{v}^2}{r} \Rightarrow \boxed{\mathrm{v} = \sqrt{\frac{GM}{r}}}$$

$$T = \frac{2\pi r}{\mathrm{v}} = \frac{2\pi r}{\sqrt{GM/r}}$$

$$\boxed{T = \frac{2\pi r^{3/2}}{\sqrt{GM}}}$$

Orbital period

Example

Venus rotates slowly about its axis, the period being 243 days. The mass of Venus is 4.87×10^{24} kg. Determine the radius for a synchronous satellite in orbit about Venus.

<u>Solution:</u>

Given: $M_V = 4.87 \text{ x}10^{24}$ kg, $T_V = 243$ days

Recognize: Synchronous means that the period of the satellite equals the period of Venus, $T_s = T_V$

Convert T_V to seconds and find r_s

$$T_V = 243\text{ days}\left(\frac{24\text{ hr}}{1\text{ day}}\right)\left(\frac{60\text{ min}}{1\text{ hr}}\right)\left(\frac{60\text{ sec}}{1\text{ min}}\right) = 2.10\text{x}10^7\text{ s}$$

$$T_S = \frac{2\pi r_s^{3/2}}{\sqrt{GM_V}} \Rightarrow r_s^{3/2} = \frac{T_s\sqrt{GM_V}}{2\pi}$$

$$r_s^{3/2} = \frac{(2.10\text{x}10^7\text{ s})\sqrt{(6.6726\text{x}10^{-11}\ \frac{\text{Nm}^2}{\text{kg}^2})(4.87\text{x}10^{24}\text{ kg})}}{2\pi}$$

$$= 6.025\text{ x }10^{13}\text{ m}^{3/2} \Rightarrow \boxed{r_s = 1.54\text{ x }10^9\text{ m}}$$

Compare this to the radius of Venus: $6.05\text{x}10^6$ m