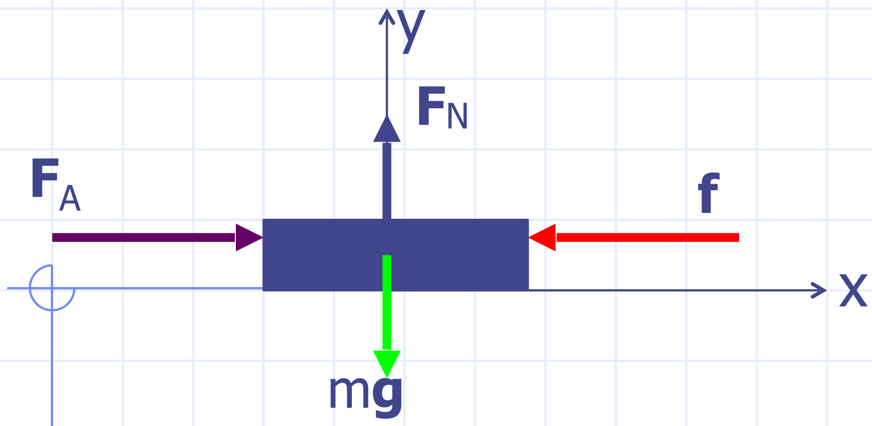


Apply Newton's 2nd Law

$$\sum F_y = F_N - mg = ma_y = 0$$

$$F_N = mg$$

$$\sum F_x = F_A - f = ma_x$$

- If applied force is small, book does not move (static), a_x=0, then f=f_s

$$F_A = f_S$$

- Increase applied force, book still does not move
- Increase F_A more, now book moves, $a_x \neq 0$

$$F_A - f_S = ma_x \Rightarrow F_A = f_S + ma_x > f_S$$

- There is some maximum static frictional force, f_s^{max}. Once the applied force exceeds it, the book moves

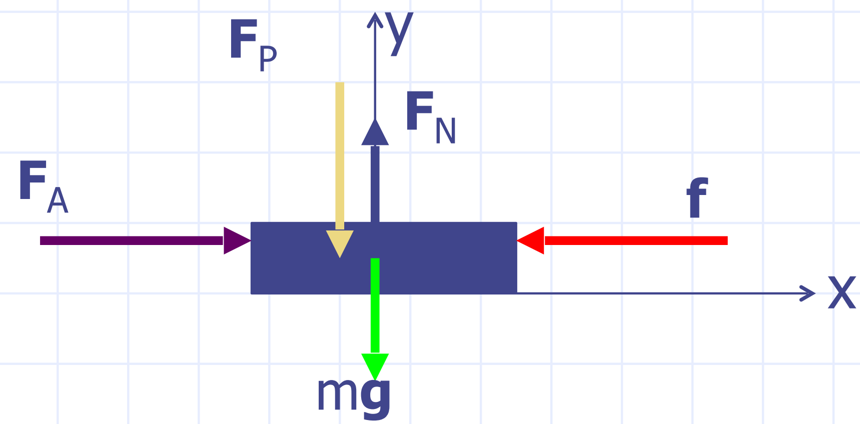
$$|\vec{f}_s^{\max}| = \mu_s |\vec{F}_N|$$

Magnitudes not vectors

- μ_s is the coefficient of static friction, it is a dimensionless number, different for each surface-object pair (wood-wood, wood-metal); also depends on surface preparation
- μ_s does not depend on the mass or surface area of the object
- Has value: $0 < \mu_s < 1.5$
- If no applied vertical force

$$f_S^{\max} = \mu_S mg$$

- Push down on book



- Apply Newton's 2nd Law

$$\sum F_y = F_N - mg - F_P = ma_y = 0$$

$$F_N = mg + F_P$$

$$f_S^{\max} = \mu_S F_N = \mu_S (mg + F_P)$$

- What is F_A needed just to start book moving?

$$\sum F_x = F_A - f_S = 0 \Rightarrow F_A = f_S^{\max} = \mu_S (mg + F_P)$$

When an Object is Moving?

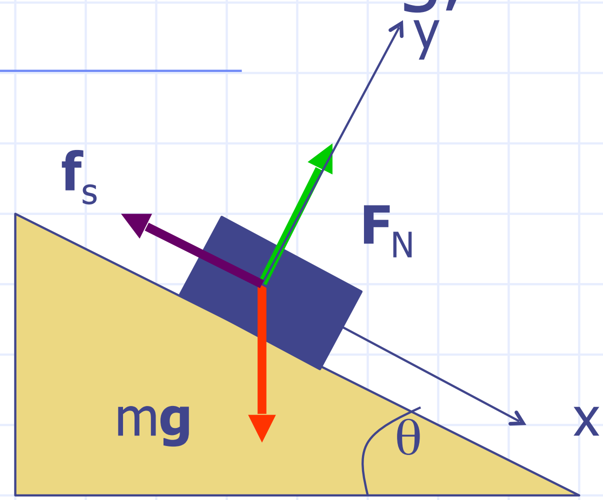
- f_s^{max} is exceeded so the object can move, but friction force is still being applied.
- However, less force is needed to keep an object moving (against friction) than to get it started
- We define kinetic friction $$f_k = \mu_k F_N$$
- μ_k is the coefficient of kinetic friction, similar to μ_s but always less than μ_s

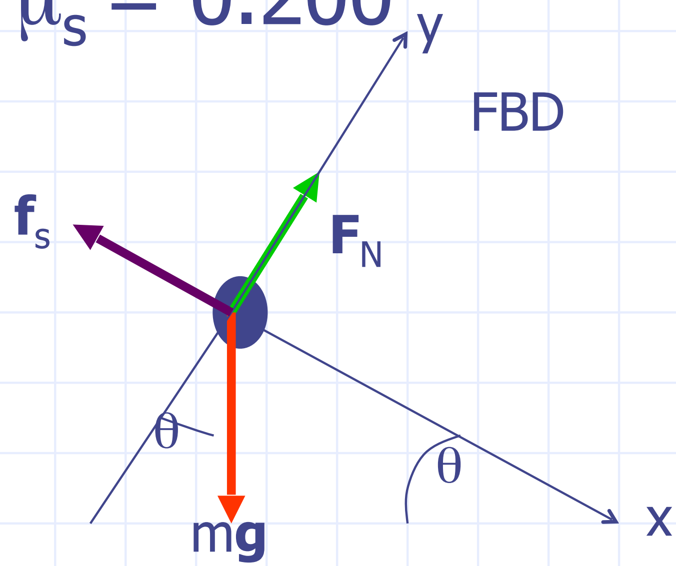
• Now, let's consider incline plane problem, but with friction (but first do a simpler example)

Example Problem

When you push a 1.80-kg book on a tabletop, it takes 2.25 N to start the book sliding. Once it is sliding, however, it takes 1.50 N to keep the book moving with constant speed. What are the coefficients of static and kinetic friction between the book and the table top?

- Book is at rest
- m = 1.00 kg, θ = 10.0°, μ_s = 0.200





$$\sum F_y = F_N - mg\cos\theta = 0$$

$$F_N = mg\cos\theta$$

$$= (1.0\text{kg})(9.80\tfrac{\text{m}}{\text{s}^2})(\cos 10.0^o)$$

$$= \boxed{9.65\text{ N}}$$

$$\sum F_x = mg\sin\theta - f_S = 0$$

$$f_S = mg\sin\theta$$

$$= (1.00\text{kg})(9.80\tfrac{\text{m}}{\text{s}^2})(\sin 10.0^o)$$

$$= \boxed{1.70\text{ N}}$$

- Book can move (slide) if $mg\sin\theta \geq f_s^{max}$

- What is f_s^{max}?

$$f_S^{\max} = \mu_S F_N = (0.200)(9.65\text{ N})$$

$$= 1.93\text{ N} > f_S$$

Book does not move.

- What angle is needed to cause book to slide?

$$mg\sin\theta \geq f_S^{\max}$$

$$mg\sin\theta \geq \mu_S F_N$$

$$mg\sin\theta \geq \mu_S mg\cos\theta$$

$$\tan\theta \geq \mu_S$$

$$\theta \geq \tan^{-1}(\mu_S)$$

$$\geq \boxed{11.3^\circ}$$

- As θ is increased, F_N decreases, therefore f_s^{max} decreases

- Once book is moving, we need to use the kinetic coefficient of friction
- Lets take, $\theta = 15.0°$ and $\mu_k = 0.150 < \mu_s$

$$\sum F_x = mg\sin\theta - f_k = ma_x$$

$$= mg\sin\theta - \mu_k mg\cos\theta = ma_x$$

$$\text{or } a_x = g(\sin\theta - \mu_k\cos\theta)$$

$$= (9.80\,\tfrac{\text{m}}{\text{s}^2})(\sin 15.0^o - 0.150\cos 15.0^o)$$

$$= \boxed{1.12\,\tfrac{\text{m}}{\text{s}^2}}$$

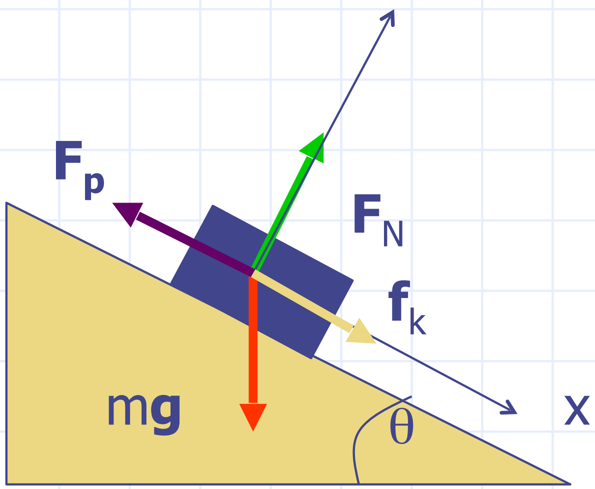
Example Problem

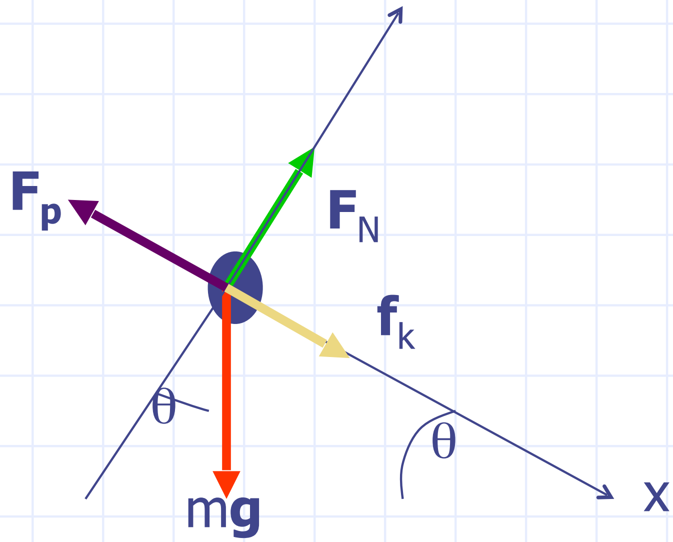
A skier is pulled up a slope at a constant velocity by a tow bar. The slope is inclined at 25.0° with respect to the horizontal. The force applied to the skier by the tow bar is parallel to the slope. The skier's mass is 55.0 kg , and the coefficient of kinetic friction between the skis and the snow is 0.120. Find the magnitude of the force that the tow bar exerts on the skier.

- Given: m = 55.0 kg, μ_k = 0.120, θ = 25.0°
- Infer: since velocity is constant, a_x=0; also a_y=0 since skier remains on slope $\Rightarrow$ equilibrium

$$\sum \vec{F} = 0 \Rightarrow \sum F_x = 0, \sum F_y = 0$$

Draw FBD, apply Newton's 2nd Law





$$\sum F_y = 0$$

$$F_N - mg\cos\theta = 0$$

$$F_N = mg\cos\theta$$

$$\sum F_x = 0$$

$$f_k - F_P + mg\sin\theta = 0$$

$$F_P = f_k + mg\sin\theta$$

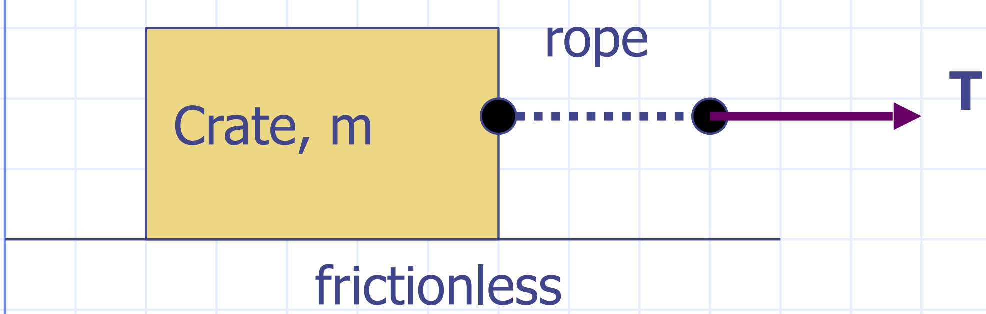
$$= \mu_k F_N + mg\sin\theta$$

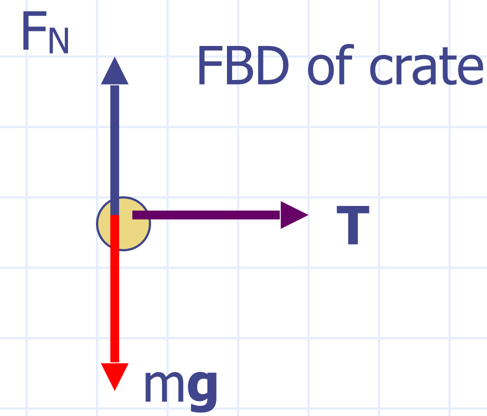
$$= \mu_k mg\cos\theta + mg\sin\theta$$

$$= mg(\mu_k \cos\theta + \sin\theta)$$

$$= \boxed{286\ \text{N}}$$

The Tension Force

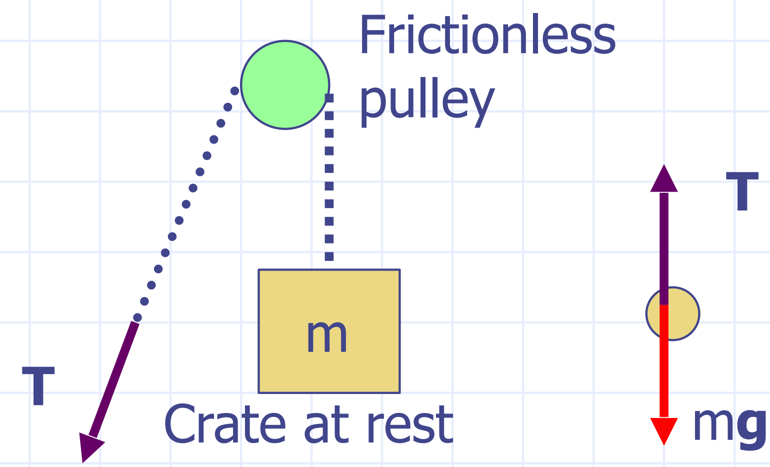




- Assume rope is massless and taut

FBD of rope

$$\sum F_x = T - T = 0$$



$$\sum F_y = T - mg = 0$$

$$T = mg$$

- Like the normal force, the friction and tension forces are all manifestations of the electromagnetic force
- They all are the result of attractive (and repulsive) forces of atoms and molecules within an object (normal and tension) or at the interface of two objects

Applications of Newton's 2nd Law

- Equilibrium – an object which has zero acceleration, can be at rest or moving with constant velocity

$$\sum \vec{F} = 0 \Rightarrow \sum F_x = 0, \sum F_y = 0$$

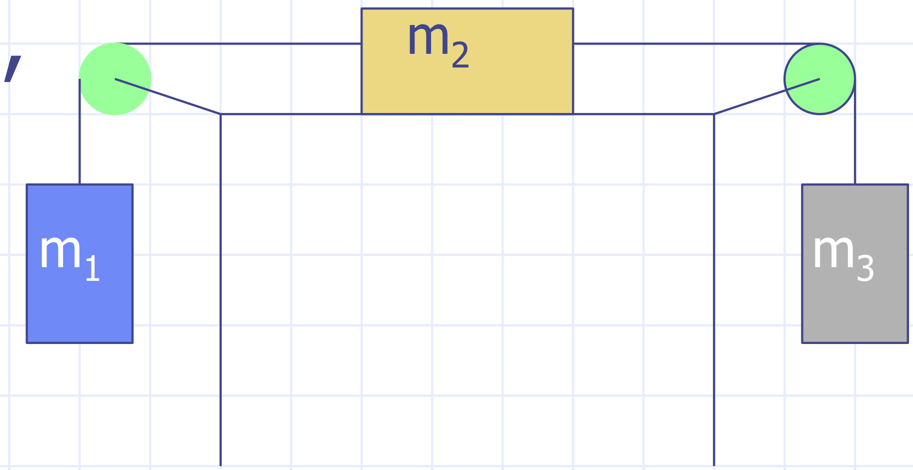
- Example: book at rest on an incline with friction
- Non-equilibrium – the acceleration of the object(s) is non-zero

$$\sum \vec{F} = m\vec{a} \Rightarrow \sum F_x = ma_x, \sum F_y = ma_y$$

Example Problem

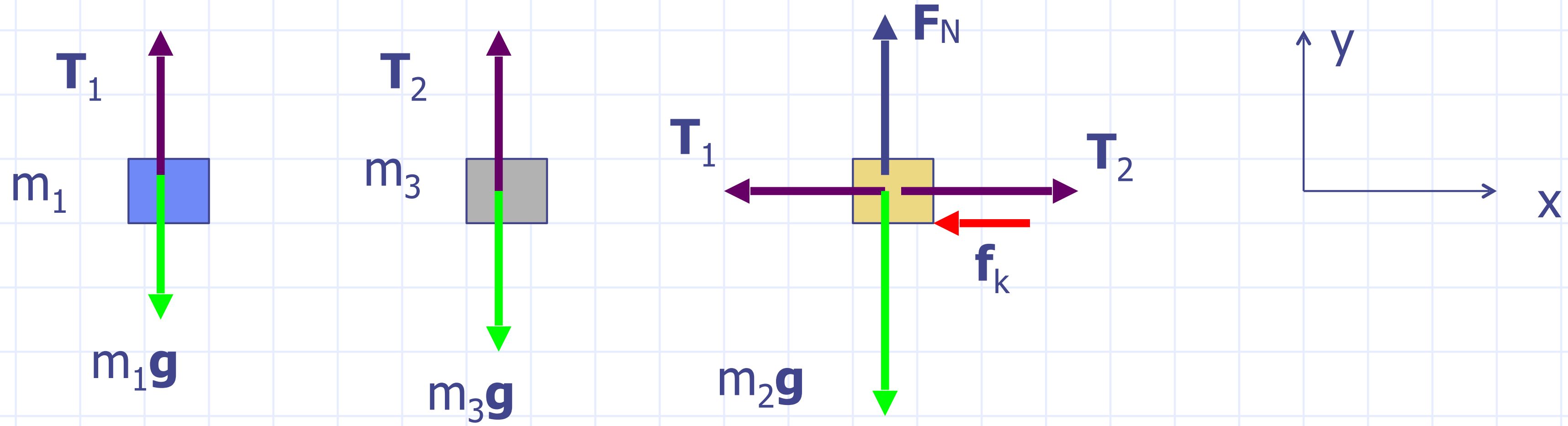
Three objects are connected by strings that pass over massless and frictionless pulleys. The objects move and the coefficient of kinetic friction between the middle object and the surface of the table is 0.100 (the other two being suspended by strings). (a) What is the acceleration of the three objects? (b) What is the tension in each of the two strings?

- Given: m_1 = 10.0 kg, m_2=80.0 kg, m_3=25.0 kg, μ_k=0.100



- Find: a_1, a_2, a_3, T_1, and T_2

Solution: 1) Draw free-body diagrams



2) Apply Newton's 2nd Law to each object

$$\sum F_{y1} = m_1 a_{y1}$$

$$T_1 - m_1 g = m_1 a_{y1}$$

$$\boxed{T_1 = m_1 (a_{y1} + g)}$$

$$\sum F_{y3} = m_3 a_{y3}$$

$$T_2 - m_3 g = m_3 a_{y3}$$

$$\boxed{T_2 = m_3 (a_{y3} + g)}$$

$$\sum F_{x2} = m_2 a_{x2}$$

$$\boxed{T_2 - T_1 - f_k = m_2 a_{x2}}$$

$$\sum F_{y2} = 0$$

$$F_N - m_2 g = 0$$

$$\boxed{F_N = m_2 g}$$

Also, $a_{x1}=0$, $a_{x3}=0$, $a_{y2}=0$

$$f_k = \mu_k F_N = \mu_k m_2 g$$

$$T_2 - T_1 - \mu_k m_2 g = m_2 a_{x2}$$

$$T_1 = m_1(a_{y1} + g)$$

$$T_2 = m_3(a_{y3} + g)$$

- Three equations, but five unknowns: a_{y1}, a_{y3}, a_{x2}, T_1, and T_2

- But, $a_{y1} = a_{x2} = -a_{y3} = a$

- Substitute 2nd and 3rd equations into the 1st equation

$$m_3(-a + g) - m_1(a + g) - \mu_k m_2 g = m_2 a$$

$$-m_3 a + m_3 g - m_1 a$$
$$- m_1 g - \mu_k m_2 g - m_2 a = 0$$

$$a(-m_3 - m_1 - m_2) + g(m_3 - m_1 - \mu_k m_2) = 0$$

$$a(m_1 + m_2 + m_3) = g(m_3 - m_1 - \mu_k m_2)$$

$$a = g\frac{(m_3 - m_1 - \mu_k m_2)}{(m_1 + m_2 + m_3)}$$

$$= 9.80\left[\frac{25.0 - 10.0 - 80.0(0.100)}{10.0 + 80.0 + 25.0}\right]$$

$$= \boxed{0.60\,\tfrac{\mathrm{m}}{\mathrm{s}^2} = a_{y1} = a_{x2} = -a_{y3}}$$

$$T_1 = m_1(a+g) = 10.0(0.60+9.80)$$

$$= \boxed{104\ \text{N}}$$

$$T_2 = m_3(g-a) = 25.0(9.80-0.60)$$

$$= \boxed{230\ \text{N}}$$

Example (simple)

- Pulling up on a rope, you lift a 4.35-kg bucket of water from a well with an acceleration of 1.78 m/s^2. What is the tension in the rope?