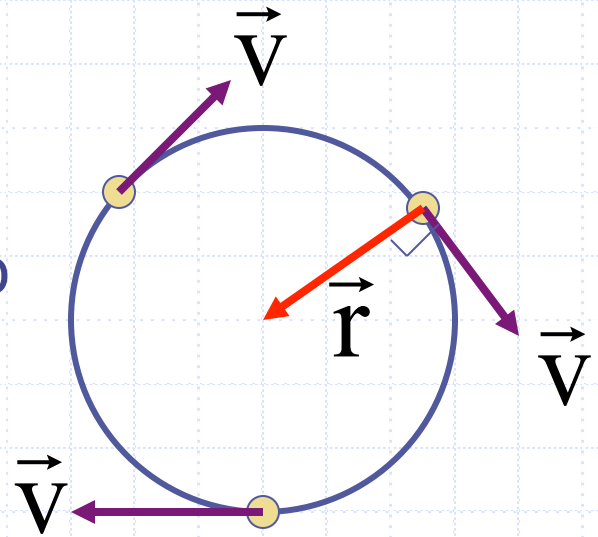
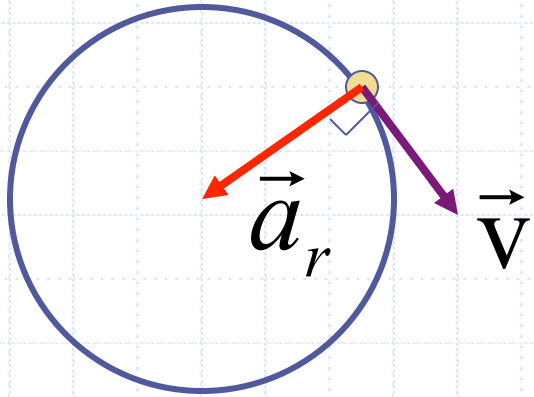
Uniform Circular Motion

- An object moving on a circular path of radius r at a constant speed v
- As motion is not on a straight line, the direction of the velocity vector is not constant
- The motion is circular
- Compare to:
  - 1D – straight line
  - 2D – parabola
- Velocity vector is always tangent to the circle
- Velocity direction constantly changing, but magnitude remains constant

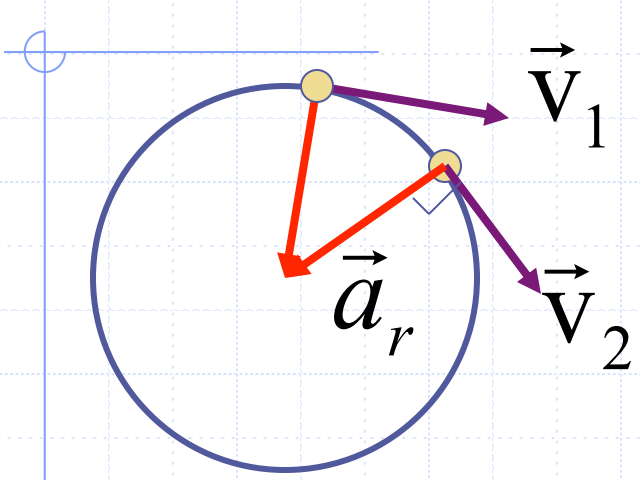


- Vectors **r** and **v** are always perpendicular
- Since the velocity direction always changes, this means that the velocity is not constant (though speed is constant), therefore the object is accelerating
- The acceleration $\mathbf{a}_r$ points radially inward. Like velocity, its direction changes, therefore the acceleration is not constant (though its magnitude is)

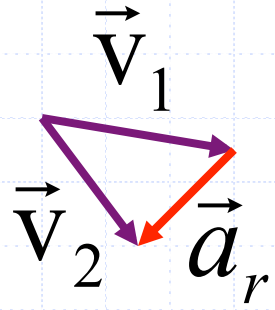


- Vectors $\mathbf{a}_r$ and **v** are also perpendicular
- The speed does not change, since $\mathbf{a}_r$ acceleration has no component along the velocity direction

- Why is the acceleration direction radially inward?



$\Rightarrow$



Since

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

- This radial acceleration is called the centripetal acceleration

$$a_r = \frac{v^2}{r} \qquad \text{(eq. 4.32)}$$

- Time to complete a full orbit

$$D = 2\pi r = \text{circumference}$$

$$T = \frac{\text{circumference}}{\text{speed}} = \frac{2\pi r}{\text{v}} = \text{Period}$$

- The Period T is the time (in seconds) for the object to make one complete orbit or cycle

- Find some useful relations for v and a_r in terms of T

$$\boxed{\text{v} = \frac{2\pi r}{T}}$$

$$a_r = \frac{\text{v}^2}{r} = \left(\frac{2\pi r}{T}\right)^2 \frac{1}{r} = \boxed{\frac{4\pi^2 r}{T^2}} = a_r$$