Chapter 21: Superposition, Interference, and Standing Waves

- In Chap. 20, we considered the motion of a *single* wave in space and time

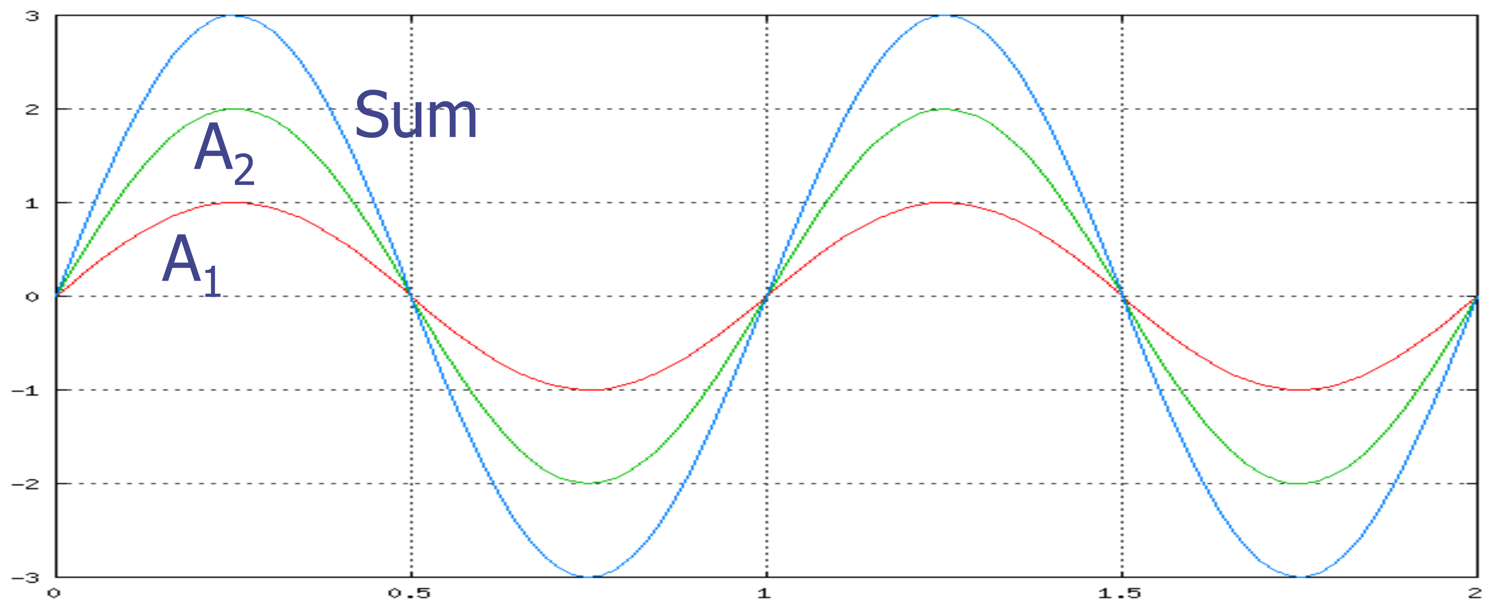
- What if there are two waves present simultaneously – in the same place and time

- Let the first wave have λ_1 and T_1, while the second wave has λ_2 and T_2

- The two waves (or more) can be added to give a *resultant wave* $\rightarrow$ this is the *Principle of Linear Superposition*

• Consider the simplest example: $\lambda_1 = \lambda_2$

- Since both waves travel in the same medium, the wave speeds are the same, then $T_1=T_2$
- We make the additional condition, that the waves have the same *phase* – i.e. they start at the same time → *Constructive Interference*
- The waves have $A_1=1$ and $A_2=2$. Here the sum of the amplitudes $A_{sum}=A_1+A_2 = 3$ ($y=y_1+y_2$)



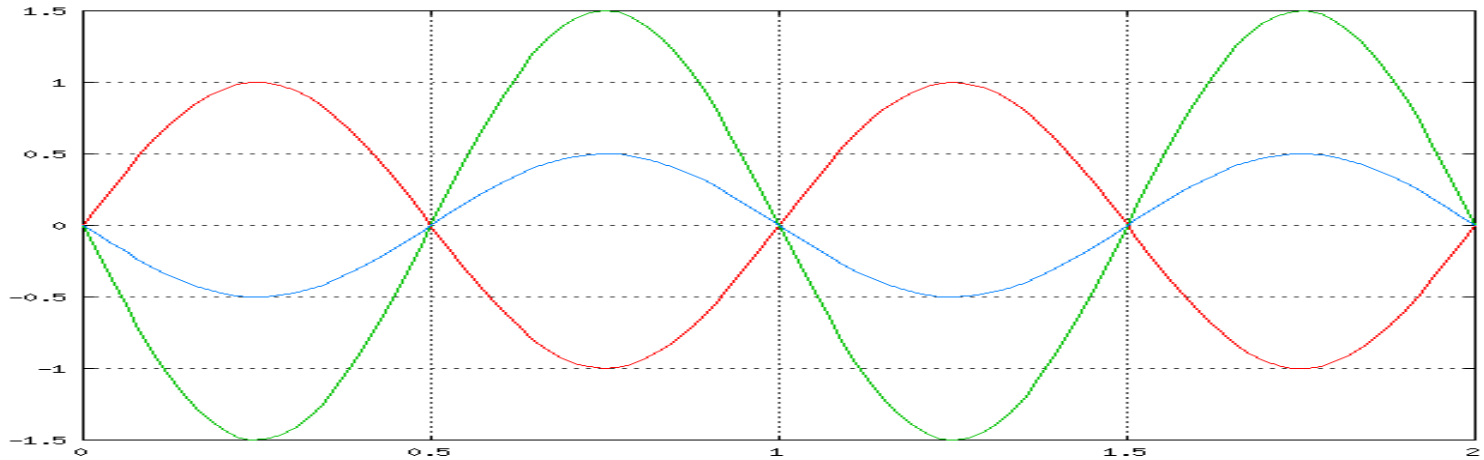
❑ If the waves ($\lambda_1 = \lambda_2$ and $T_1 = T_2$) are exactly out of phase, i.e. one starts a half cycle later than the other → *Destructive Interference*

- If $A_1 = A_2$, we have complete cancellation: $A_{sum} = 0$

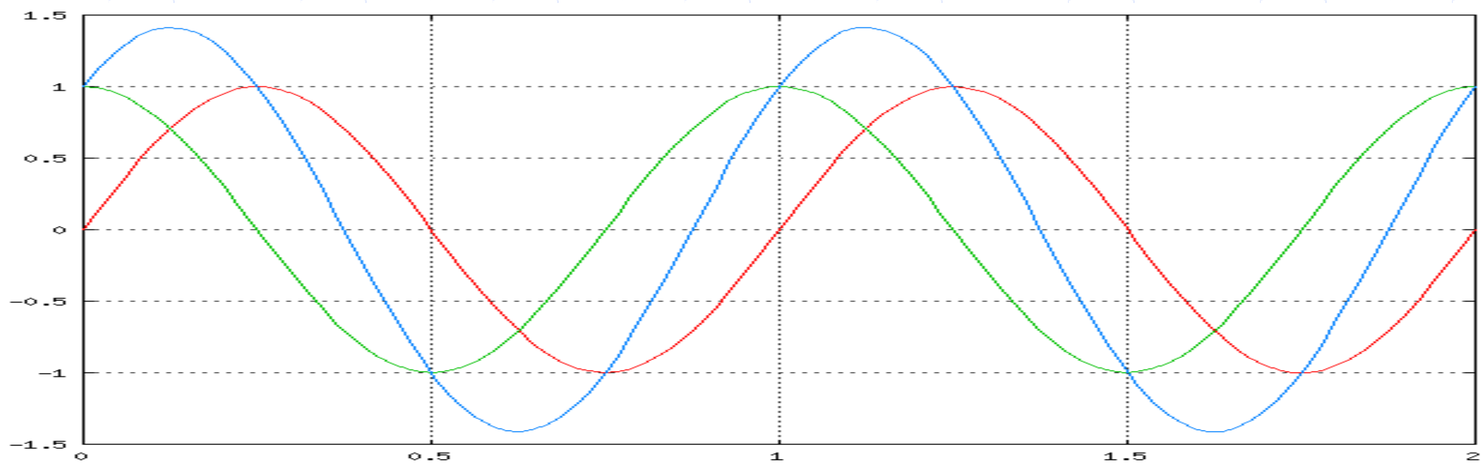
$y = y_1 + y_1 = 0$

❑ These are special cases. Waves may have different wavelengths, periods, and amplitudes and may have some fractional phase difference.

- Here are a few more examples: exactly out of phase (π), but different amplitudes



- Same amplitudes, but out of phase by ($\pi/2$)



Example Problem

Speakers A and B are vibrating in phase. They are directed facing each other, are 7.80 m apart, and are each playing a 73.0-Hz tone. The speed of sound is 343 m/s. On a line between the speakers there are three points where constructive interference occurs. What are the distances of these three points from speaker A?

Solution:

Given: $f_A = f_B = 73.0$ Hz, L=7.80 m, v=343 m/s

$$\lambda = \mathrm{v}T = \frac{\mathrm{v}}{f} = \frac{343\ \mathrm{m/s}}{73.0\ \mathrm{Hz}} = 4.70\ \mathrm{m}$$

$$L = \lambda + 2x$$

x is the distance to the first constructive interference point

$$x = \frac{L}{2} - \frac{\lambda}{2}$$

The next point (node) is half a wave-length away.

$$x = \frac{L}{2} - n\frac{\lambda}{2}$$

Where n=0,1,2,3,... for all nodes

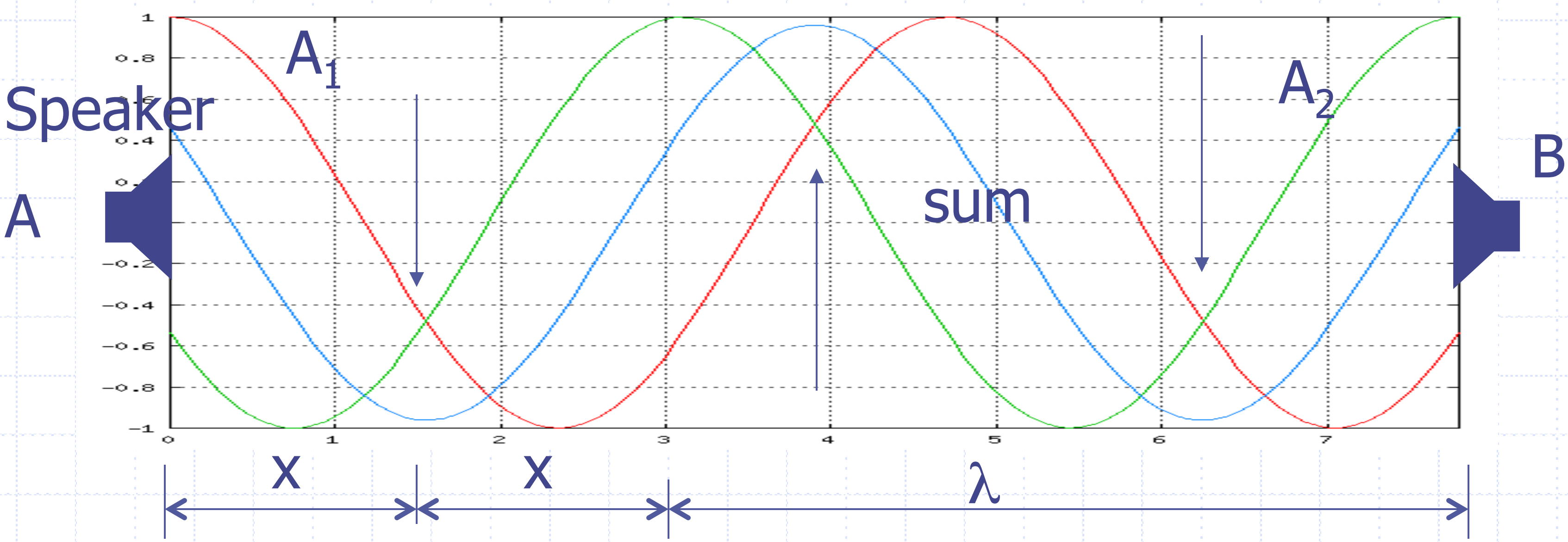
$$n = 0:\ x = \frac{7.8}{2} - 0\frac{4.70}{2} = \boxed{3.9\ \mathrm{m}}$$

$$n = 1:\ x = \frac{7.8}{2} - 1\frac{4.70}{2} = \boxed{1.55\ \mathrm{m}}$$

$$n = 2:\ x = \frac{7.8}{2} - 2\frac{4.70}{2} = -0.8\ \mathrm{m}$$

Behind speaker A

$$n = -1:\ x = \frac{7.8}{2} - (-1)\frac{4.70}{2} = \boxed{6.25\ \mathrm{m}}$$



Beats

- Different waves usually don't have the same frequency. The frequencies may be much different or only slightly different.
- If the frequencies are only *slightly* different, an interesting effect results → the ***beat frequency.***
- Useful for tuning musical instruments.
- If a guitar and piano, both play the same note (same frequency, $f_1=f_2$) → *constructive interference*
- If f_1 and f_2 are only slightly different, constructive and destructive interference occurs

- The beat frequency is

$$f_b = |f_1 - f_2| \quad \textbf{or}$$

$$\frac{1}{T_b} = \left|\frac{1}{T_1} - \frac{1}{T_2}\right|$$ In terms of periods

$$\textbf{as } f_2 \to f_1, \quad f_b \to 0$$

- The frequencies become ``tuned”

Example Problem

When a guitar string is sounded along with a 440-Hz tuning fork, a beat frequency of 5 Hz is heard. When the same string is sounded along with a 436-Hz tuning fork, the beat frequency is 9 Hz. What is the frequency of the string?

Solution:

Given: f_{T1}=440 Hz, f_{T2}=436 Hz, f_{b1}=5 Hz, f_{b2}=9 Hz

But we don't know if frequency of the string, f_s, is greater than f_{T1} and/or f_{T2}. Assume it is.

$$f_{b1} = f_s - f_{T1} \text{ and } f_{b2} = f_s - f_{T2} \Rightarrow$$

$$f_s = f_{b1} + f_{T1} = 5 + 440 = \boxed{445\text{ Hz}}$$

$$f_s = f_{b2} + f_{T2} = 9 + 436 = \boxed{445\text{ Hz}}$$

If we chose f_s smaller

$$f_{b1} = f_{T1} - f_s \text{ and } f_{b2} = f_{T2} - f_s \Rightarrow$$

$$f_s = f_{T1} - f_{b1} = 440 - 5 = \cancel{435\text{ Hz}}$$

$$f_s = f_{T2} - f_{b2} = 436 - 9 = \cancel{427\text{ Hz}}$$

$\neq$

Standing Waves

- A *standing wave* is an interference effect due to two overlapping waves
  - transverse – wave on guitar string, violin, …
  - longitudinal – sound wave in a flute, pipe organ, other wind instruments,…
- The length (dictated by some physical constraint) of the wave is some multiple of the wavelength
- You saw this in lab a few weeks ago
- Consider a *transverse* wave (f_1, T_1) on a string of length L fixed at both ends.

- If the speed of the wave is v (not the speed of sound in air), the time for the wave to travel from one end to the other and back is $2L/\mathrm{v}$
- If this time is equal to the period of the wave, T_1, then the wave is a *standing wave*

$$T_1 = \frac{1}{f_1} = \frac{2L}{\mathrm{v}} \Rightarrow \boxed{f_1 = \frac{\mathrm{v}}{2L}} = \frac{\mathrm{v}}{\lambda_1} \Rightarrow \lambda_1 = 2L$$

- Therefore the length of the wave is half of a wavelength or a half-cycle is contained between the end points
- We can also have a full cycle contained between end points

$$\lambda_2 = L \Rightarrow f_2 = \frac{\mathrm{v}}{\lambda_2} = \boxed{\frac{\mathrm{v}}{L} = f_2}$$

- Or three half-cycles

$$\lambda_3 = \tfrac{2}{3}L \Rightarrow f_3 = \frac{v}{\lambda_3} = \frac{v}{\tfrac{2}{3}L} = \boxed{\frac{3v}{2L} = f_3}$$

- Or *n* half-cycles

$$\boxed{f_n = n\left(\frac{v}{2L}\right)}, \quad n = 1, 2, 3, 4, \ldots$$

For a string fixed at both ends

- Some notation:

f_1	1st harmonic	or fundamental
$f_2 = 2f_1$	2nd	1st overtone
$f_3 = 3f_1$	3rd	2nd overtone
$f_4 = 4f_1$	4th	3rd overtone

- The zero amplitude points are called *nodes*; the maximum amplitude points are the *antinodes*

Longitudinal Standing Waves

- Consider a tube with both ends opened
- If we produce a sound of frequency f_1 at one end, the air molecules at that end are free to vibrate and they vibrate with f_1
- The amplitude of the wave is the amplitude of the vibrational motion (SHM) of the air molecule – changes in air density
- Similar to the transverse wave on a string, a standing wave occurs if the length of the tube is a half-multiple of the wavelength of the wave

- For the first harmonic (fundamental), only half of a cycle is contained in the tube

$$f_1 = \frac{v}{2L}$$

- Following the same reasoning as for the transverse standing wave, all of the harmonic frequencies are

$$\boxed{f_n = n\left(\frac{v}{2L}\right), \quad n = 1, 2, 3, ...}$$

Open-open tube

- Identical to transverse wave, except number of nodes is different

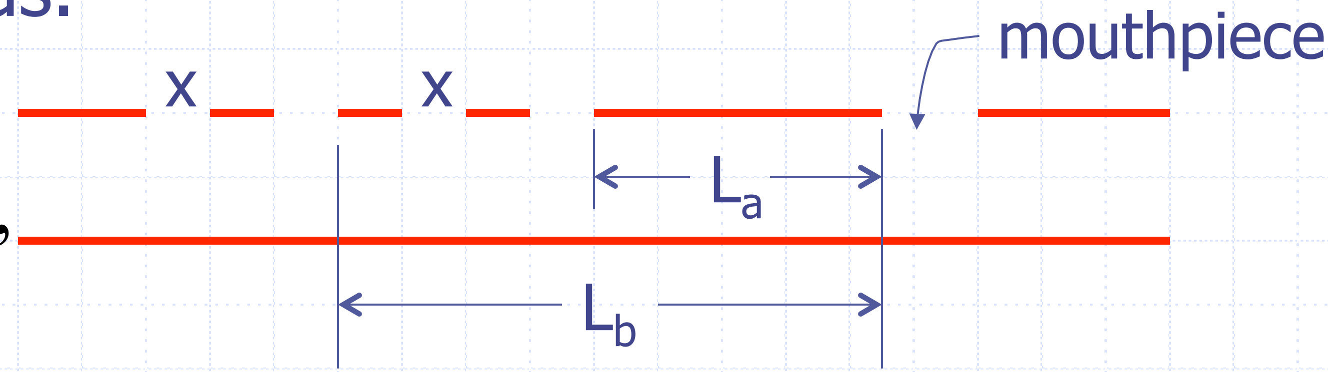
$$\#\,\text{nodes} = n - 1$$

string

$$\#\,\text{nodes} = n$$

Open-open tube

- An example is a flute. It is a tube which is open at both ends.



$$f_a = \frac{v}{2L_a},$$

$$f_b = \frac{v}{2L_b} < f_a$$

- We can also have a tube which is closed at one end and opened at the other (open-closed)
- At the closed end, the air molecules can not vibrate – the closed end must be a ``node”
- The open end must be an anti-node

- The ``distance” between a node and the next adjacent anti-node is 1/4 of a wavelength. Therefore the fundamental frequency of the open-closed tube is

$$f_1 = \frac{\text{v}}{4L} \quad \text{since} \quad L = \lambda / 4 \ \text{ or } \lambda = 4L$$

- The next harmonic does not occur for 1/2 of a wavelength, but 3/4 of a wavelength. The next is at 5/4 of a wavelength – every odd 1/4 wavelength

$$\boxed{f_n = n\left(\frac{\text{v}}{4L}\right), n = 1,3,5,\ldots}$$

Open-closed

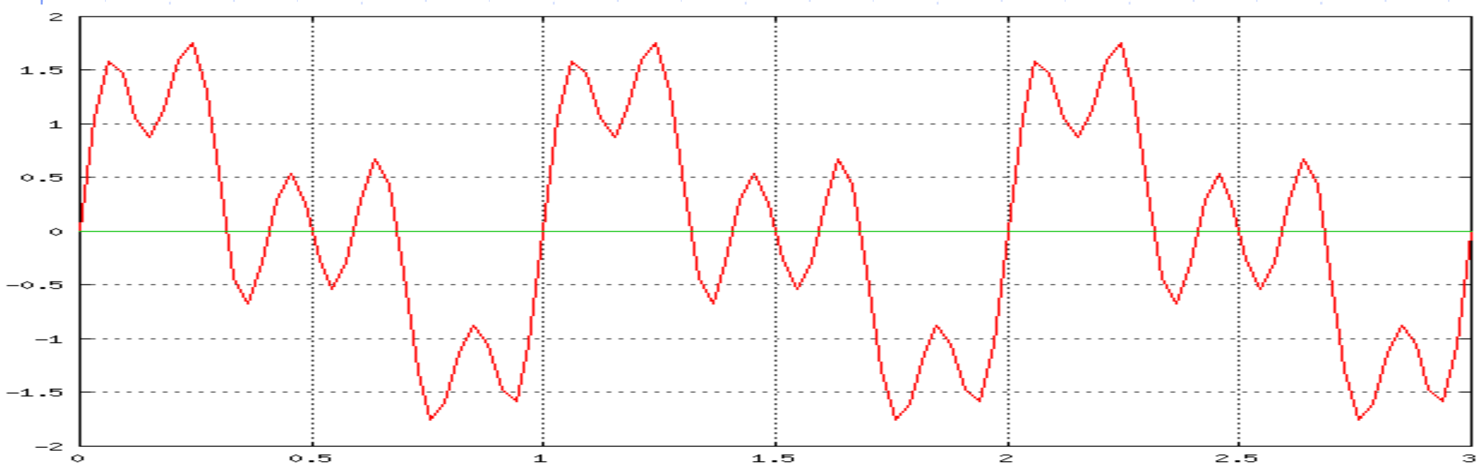
- Note that the even harmonics are missing. Also,

$$\#\text{nodes} = \frac{n-1}{2}$$

Complex (Real) Sound Waves

- Most sounds that we hear are *not* pure tones (single frequency – like the fundamental f_1 of a standing wave)
- But are superpositions of many frequencies with various amplitudes
- For example, when a note (tone, frequency) is played on a musical instrument, we actually hear all of the harmonics (f_1, f_2, f_3, …), but usually the amplitudes are decreased for the higher harmonics
- This is what gives each instrument it's unique sound

❑ For example, the sound of a piano is dominated by the 1st harmonic while for the violin, the amplitudes of the 1st, 2nd, and 5th harmonic are nearly equal – gives it a rich sound



Violin wave form

Summary

String fixed at both ends and the open-open tube

$$f_n = n\left(\frac{v}{2L}\right), \quad n = 1, 2, 3, \ldots.$$

Open-closed tube

$$f_n = n\left(\frac{v}{4L}\right), \quad n = 1,3,5,\ldots$$

Example Problem

- A tube with a cap on one end, but open at the other end, produces a standing wave whose fundamental frequency is 130.8 Hz. The speed of sound is 343 m/s. (a) If the cap is removed, what is the new fundamental frequency? (b) How long is the tube?

Solution:

Given: f_1^{oc}=130.8 Hz, n=1, v=343 m/s

$$f_n^{oc} = n\left(\frac{v}{4L}\right) \qquad f_n^{oo} = n\left(\frac{v}{2L}\right)$$

(a) We don't need to know v or L, since they are the same in both cases. Solve each equation for v/L and set equal

$$\frac{\mathrm{v}}{L} = 4 f_1^{oc}, \quad \frac{\mathrm{v}}{L} = 2 f_1^{oo} \quad \Rightarrow \; 4 f_1^{oc} = 2 f_1^{oo}$$

$$f_1^{oo} = 2 f_1^{oc} = 2(130.8\ \mathrm{Hz}) = \boxed{261.6\ \mathrm{Hz}}$$

(b) Can solve for L from either open-open or open-closed tubes

$$f_1^{oc} = 1\left(\frac{\mathrm{v}}{4L}\right) \Rightarrow$$

$$L = \frac{\mathrm{v}}{4 f_1^{oc}} = \frac{343\ \mathrm{m/s}}{4(130.8\ \mathrm{Hz})} = \boxed{0.6556\ \mathrm{m}}$$

$$L = \frac{\mathrm{v}}{2 f_1^{oo}} = \frac{343\ \mathrm{m/s}}{2(261.6\ \mathrm{Hz})} = 0.6556\ \mathrm{m}$$