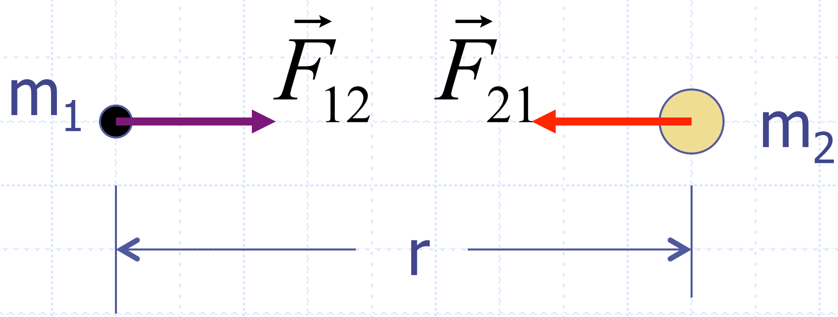
Universal Force due to Gravity (sections 6.3 and 13.3)

- Every object in the Universe exerts an attractive force on all other objects
- The force is directed along the line separating two objects
- Because of the 3rd law, the force exerted by object 1 on 2, has the same magnitude, but opposite direction, as the force exerted on 2 by 1



$$\vec{F}_{12} = -\vec{F}_{21}$$

By 3rd law

where

$$F_{12} = \frac{Gm_1m_2}{r^2}$$

And G ≡ Universal Gravitational Constant
= 6.67259×10^{-11} N m^2/kg^2

• G is a constant everywhere in the Universe, therefore it is a fundamental constant

❑ *g* is not a fundamental constant, but we can calculate it. Compare:

$$F = mg$$

and

$$F_{12} = \frac{Gm_1m_2}{r^2}$$

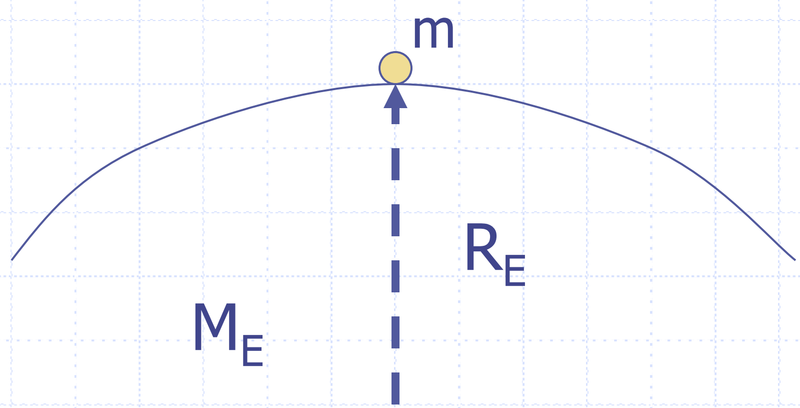
Let $m_1 = M_E$ = mass of the Earth,

$m_2 = m$ = mass of an object which is << M_E,

$r = R_E$, object is at surface of the Earth,

Set the forces equal to each other:

$$mg = \frac{GM_E m}{R_E^2}$$



$$g = \frac{GM_E}{R_E^2}$$

$$g = \frac{(6.67259\text{x}10^{-11}\ \frac{\text{Nm}^2}{\text{kg}^2})(5.9742\text{x}10^{24}\ \text{kg})}{(6.378\text{x}10^{6}\ \text{m})^2} = \boxed{9.80\ \frac{\text{m}}{\text{s}^2}}$$

❑ Weight ≠ mass

- Weight - the force exerted on an object by the Earth's gravity

$$F_G = mg = W$$

- Mass is intrinsic to an object, weight is not
- From previous page, $W=m(GM_E/R_E^2)$
  - your weight would be different on the moon
- Gravity is a very weak force, need massive objects

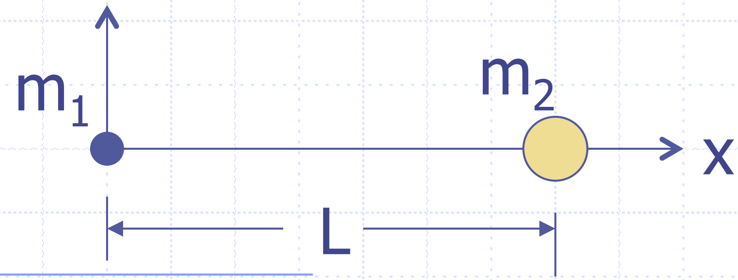
Example Problem (difficult!)

Two particles are located on the x-axis. Particle 1 has a mass of m and is at the origin. Particle 2 has a mass of 2m and is at x=+L. A third particle is placed between particles 1 and 2. Where on the x-axis should the third particle be located so that the magnitude of the gravitational force on both particles 1 and 2 doubles? Express your answer in terms of L.

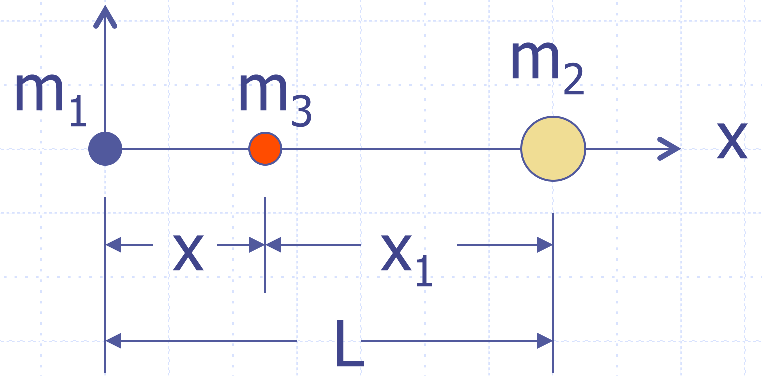
<u>Solution:</u>

Principle – universal gravitation (no Earth), $F_{12}=Gm_1m_2/r^2$

Strategy – compute forces with particles 1 and 2, then compute forces with three particles



Situation 1



Situation 2

Given: $m_1 = m$, $m_2 = 2m$, $r_{12} = L$

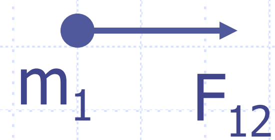
Don't know: $m_3 = ?$

Find: $x = r_{13}$ when force on 1 and 2 equals $2F_{12}$

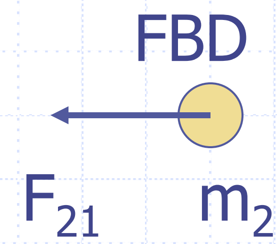
Situation 1:

FBD

$$\sum F_x = F_{12} = \frac{Gm_1 m_2}{r^2} = \frac{Gm(2m)}{L^2} = \frac{2Gm^2}{L^2}$$



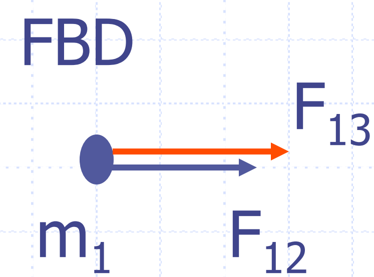
$$\sum F_x = F_{21} = -F_{12} = -\frac{2Gm^2}{L^2}$$



Situation 2:

$$\sum F_x = F_{12} + F_{13} = \frac{2Gm^2}{L^2} + \frac{Gmm_3}{x^2} = \frac{4Gm^2}{L^2}$$

Since in situation 2 the total force must be $2F_{12}$.

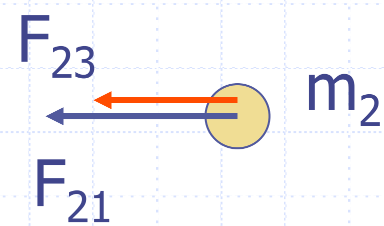


Solve for x.

$$\frac{2m}{L^2} + \frac{m_3}{x^2} = \frac{4m}{L^2} \quad \text{or} \quad \frac{m_3}{x^2} = \frac{2m}{L^2}$$

$$\frac{x^2}{m_3} = \frac{L^2}{2m} \quad \text{or} \quad \boxed{x = \pm\sqrt{\frac{m_3}{2m}}L}$$

FBD



Now consider m_2:

$$\sum F_x = F_{21} + F_{23} = -\frac{2Gm^2}{L^2} + -\frac{G(2m)m_3}{x_1^2} = -\frac{4Gm^2}{L^2}$$

$$\frac{2m}{L^2} + \frac{2m_3}{x_1^2} = \frac{4m}{L^2} \quad \text{or} \quad \frac{2m_3}{x_1^2} = \frac{2m}{L^2}$$

$$\frac{x_1^2}{2m_3} = \frac{L^2}{2m} \quad \text{or} \quad \boxed{x_1 = \pm\sqrt{\frac{m_3}{m}}L}$$

$$x + x_1 = L$$

or

$$x = L - x_1 = L \mp \sqrt{\frac{m_3}{m}} L = L\left(1 \mp \sqrt{\frac{m_3}{m}}\right)$$

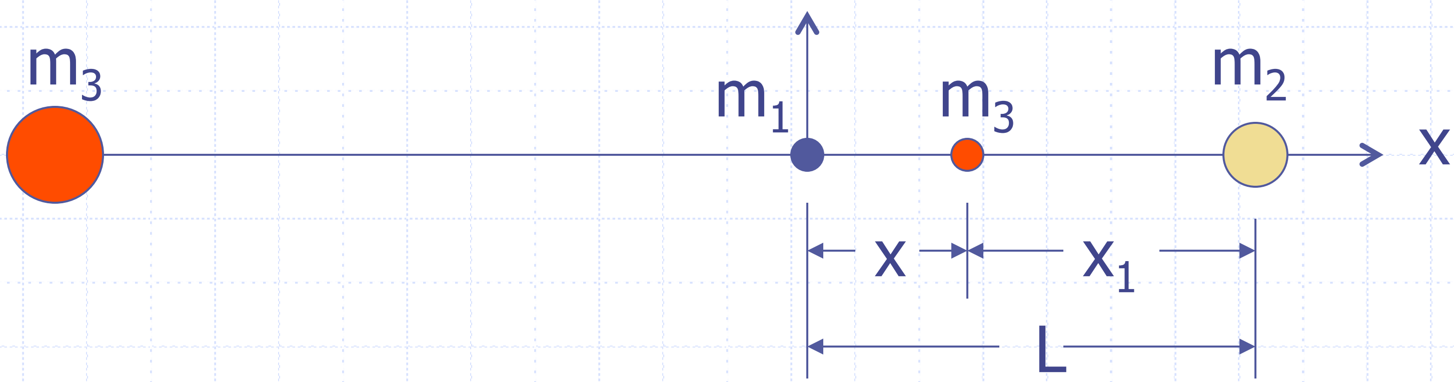
Substitute for m_3

$$x = \pm\sqrt{\frac{m_3}{2m}} L \Rightarrow \sqrt{\frac{m_3}{m}} = \mp\sqrt{2}\,\frac{x}{L}$$

$$x = L\left(1 \mp \sqrt{2}\,\frac{x}{L}\right) = L \mp \sqrt{2}x$$

$$x \pm \sqrt{2}x = x\left(1 \pm \sqrt{2}\right) = L$$

$$x = \frac{L}{1 \pm \sqrt{2}} = \boxed{0.414L \ \text{ or } -2.414L}$$



Since

$$m_3 = 0.343m \ \text{ or } \ 11.7m$$