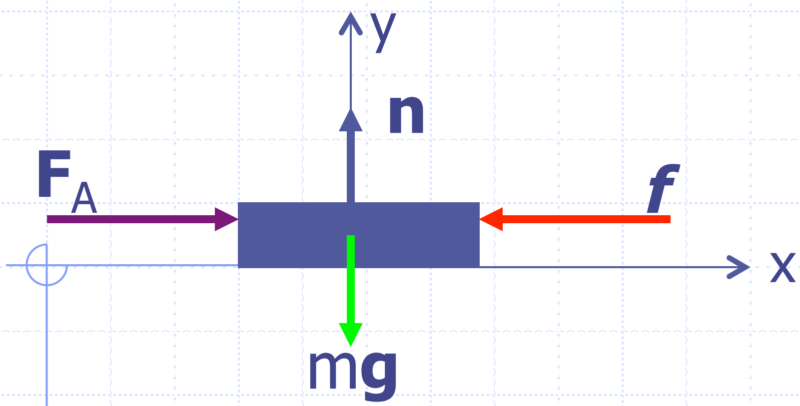


Apply Newton's 2nd Law

$$\sum F_y = n - mg = ma_y = 0$$

$$n = mg$$

$$\sum F_x = F_A - f = ma_x$$

- If applied force is small, book does not move (*static*), $a_x=0$, then $f=f_s$

$$F_A = f_S$$

- Increase applied force, book still does not move
- Increase F_A more, now book moves, $a_x \neq 0$

$$F_A - f_S = ma_x \Rightarrow F_A = f_S + ma_x > f_S$$

- There is some maximum *static* frictional force, f_s^{max}. Once the applied force exceeds it, the book moves

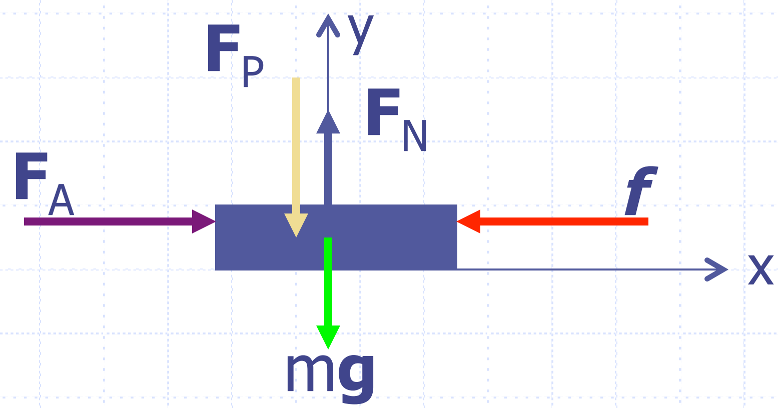
$$f_S^{\max} = \mu_S n$$

Magnitudes not vectors

- μ_s is the coefficient of *static* friction, it is a dimensionless number, different for each surface-object pair (wood-wood, wood-metal); also depends on surface preparation
- μ_s does not depend on the mass or surface area of the object
- Has value: $0 < \mu_s < 1.5$
- If no applied vertical force

$$f_S^{\max} = \mu_S mg$$

- Push down on book



- Apply Newton's 2nd Law

$$\sum F_y = F_N - mg - F_P = ma_y = 0$$

$$F_N = mg + F_P$$

$$f_S^{\max} = \mu_S F_N = \mu_S (mg + F_P)$$

- What is F_A needed just to start book moving?

$$\sum F_x = F_A - f_S = 0 \Rightarrow F_A = f_S^{\max} = \mu_S (mg + F_P)$$

When an Object is Moving?

- f_s^{max} is exceeded so the object can move, but friction force is still being applied.
- However, less force is needed to keep an object moving (against friction) than to get it started
- We define *kinetic* friction

$$f_k = \mu_k F_N$$

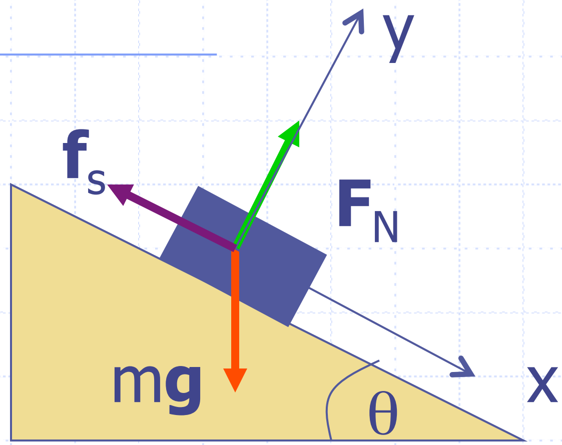
- μ_k is the coefficient of *kinetic* friction, similar to μ_S but always less than μ_S (Table 6.1)

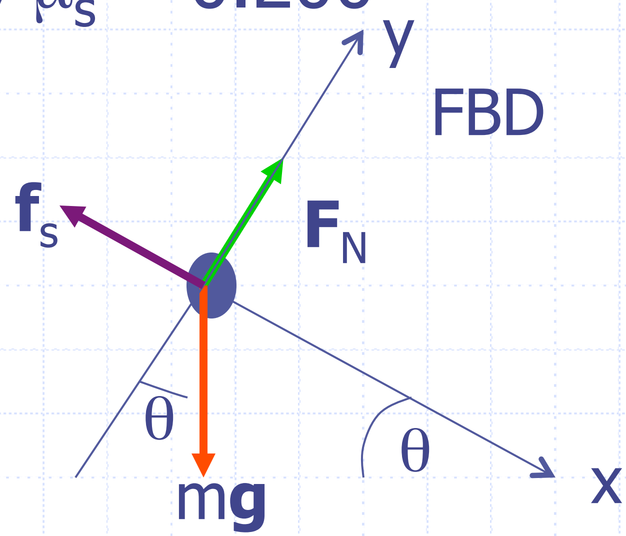
• Now, let's consider incline plane problem, but with friction (but first do a simpler example)

Example Problem

When you push a 1.80-kg book on a tabletop, it takes 2.25 N to start the book sliding. Once it is sliding, however, it takes 1.50 N to keep the book moving with constant speed. What are the coefficients of static and kinetic friction between the book and the table top?

- Book is at rest
- m = 1.00 kg, θ = 10.0°, μ_s = 0.200





$$\sum F_y = F_N - mg\cos\theta = 0$$

$$F_N = mg\cos\theta$$

$$= (1.0\text{kg})(9.80\tfrac{\text{m}}{\text{s}^2})(\cos 10.0^o)$$

$$= \boxed{9.65\text{ N}}$$

$$\sum F_x = mg\sin\theta - f_S = 0$$

$$f_S = mg\sin\theta$$

$$= (1.00\text{kg})(9.80\tfrac{\text{m}}{\text{s}^2})(\sin 10.0^o)$$

$$= \boxed{1.70\text{ N}}$$

- Book can move (slide) if mgsinθ ≥ f_s^{max}
- What is f_s^{max}?

$$f_S^{\max} = \mu_S F_N = (0.200)(9.65\ \mathrm{N})$$

$$= 1.93\ \mathrm{N} > f_S$$

Book does not move.

- What angle is needed to cause book to slide?

$$mg\sin\theta \geq f_S^{\max}$$
$$mg\sin\theta \geq \mu_S F_N$$
$$mg\sin\theta \geq \mu_S mg\cos\theta$$
$$\tan\theta \geq \mu_S$$

$$\theta \geq \tan^{-1}(\mu_S)$$
$$\geq \boxed{11.3^{\circ}}$$

- As θ is increased, F_N decreases, therefore f_s^{max} decreases

- Once book is moving, we need to use the *kinetic* coefficient of friction
- Lets take, $\theta = 15.0°$ and $\mu_k = 0.150 < \mu_s$

$$\sum F_x = mg\sin\theta - f_k = ma_x$$

$$= mg\sin\theta - \mu_k mg\cos\theta = ma_x$$

$$\text{or } a_x = g(\sin\theta - \mu_k \cos\theta)$$

$$= (9.80\,\tfrac{\text{m}}{\text{s}^2})(\sin 15.0^o - 0.150\cos 15.0^o)$$

$$= \boxed{1.12\,\tfrac{\text{m}}{\text{s}^2}}$$

Example Problem

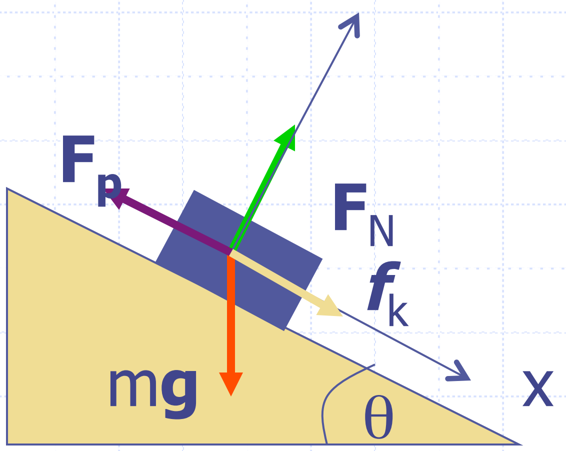
A skier is pulled up a slope at a constant velocity by a tow bar. The slope is inclined at 25.0° with respect to the horizontal. The force applied to the skier by the tow bar is parallel to the slope. The skier's mass is 55.0 kg , and the coefficient of kinetic friction between the skis and the snow is 0.120. Find the magnitude of the force that the tow bar exerts on the skier.

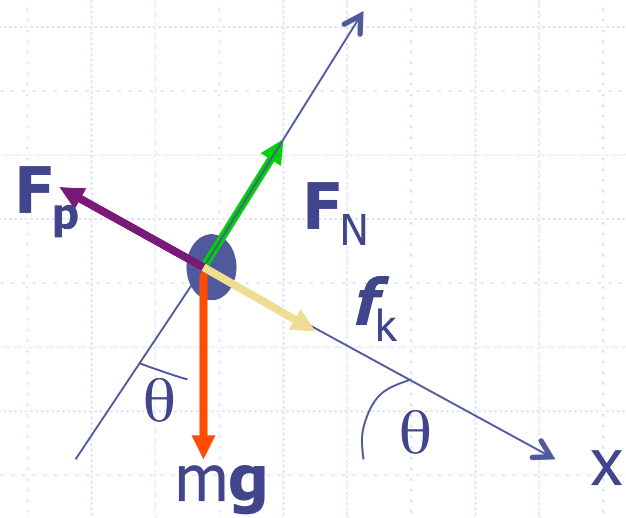
- Given: m = 55.0 kg, μ_k = 0.120, θ = 25.0°
- Infer: since velocity is constant, a_x=0; also a_y=0 since skier remains on slope

$\Rightarrow$ equilibrium

$$\sum \vec{F} = 0 \Rightarrow \sum F_x = 0, \sum F_y = 0$$

Draw FBD, apply Newton's 2nd Law





$$\sum F_y = 0$$

$$F_N - mg\cos\theta = 0$$

$$F_N = mg\cos\theta$$

$$\sum F_x = 0$$

$$f_k - F_P + mg\sin\theta = 0$$

$$F_P = f_k + mg\sin\theta$$

$$= \mu_k F_N + mg\sin\theta$$

$$= \mu_k mg\cos\theta + mg\sin\theta$$

$$= mg(\mu_k\cos\theta + \sin\theta)$$

$$= \boxed{286\ \mathrm{N}}$$