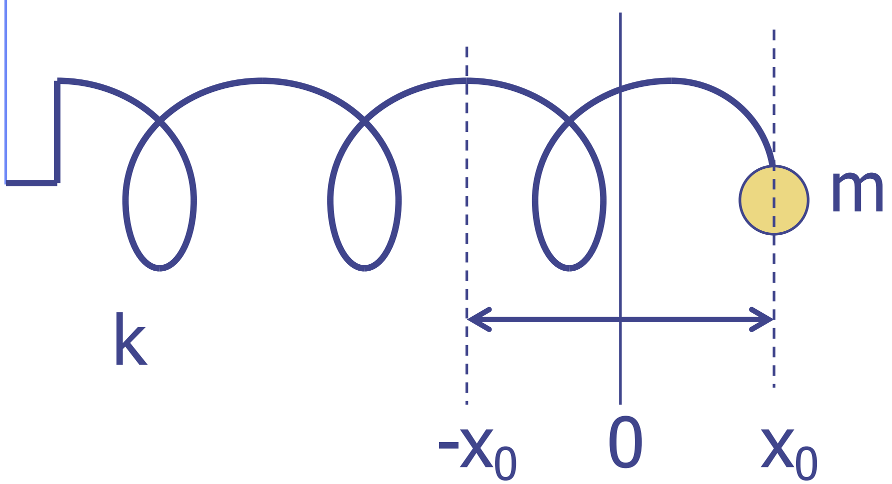
Chapter 14: Oscillatory Motion

- We continue our studies of mechanics, but combine the concepts of translational and rotational motion.
- We will revisit the *ideal spring*. In particular, we will re-examine the restoring force of the spring and its potential energy.
- We will consider the motion of a mass, attached to the spring, about its equilibrium position.
- This type of motion is applicable to many other kinds of situations: pendulum, atoms, planets, ...

Simple Harmonic Motion

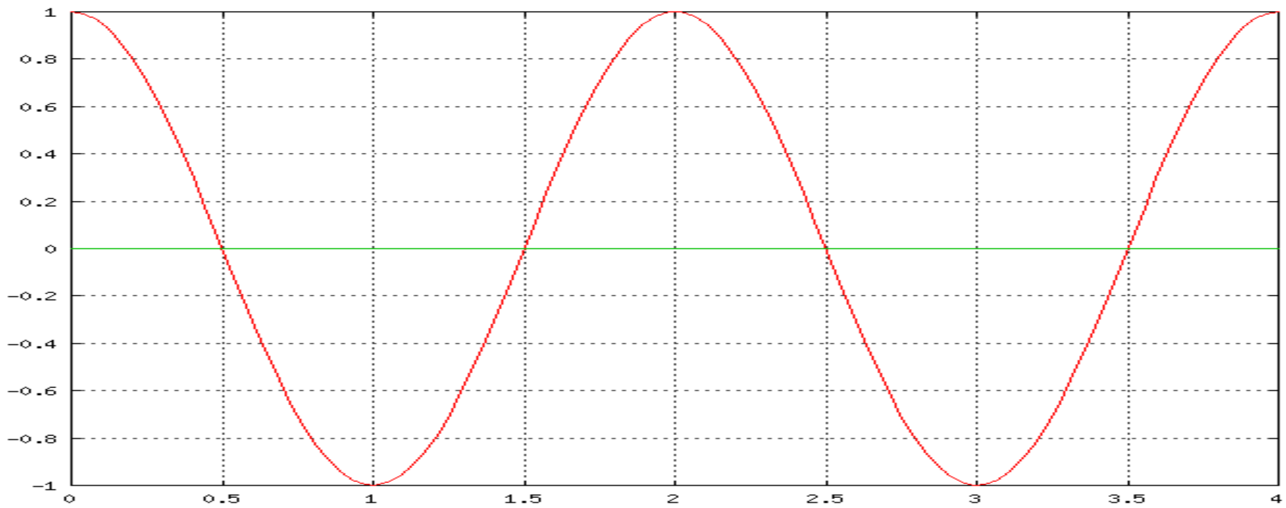
- If we add a mass m to the end of the (massless) spring, stretch it to a displacement x_0, and release it. The *spring-mass system* will oscillate from x_0 to $-x_0$ and back.



Without friction and air resistance, the oscillation would continue indefinitely

- This is Simple Harmonic Motion (SHM)

- SHM has a maximum *magnitude* of $|x_0|=A$, called the *Amplitude*



- One way to understand SHM is to reconsider the circular motion of a particle and rotational kinematics (The Reference Circle)

- The particle travels on a circle of radius $r = A$ with the line from the center to the particle making an angle θ with respect to the x-axis at some instant in time

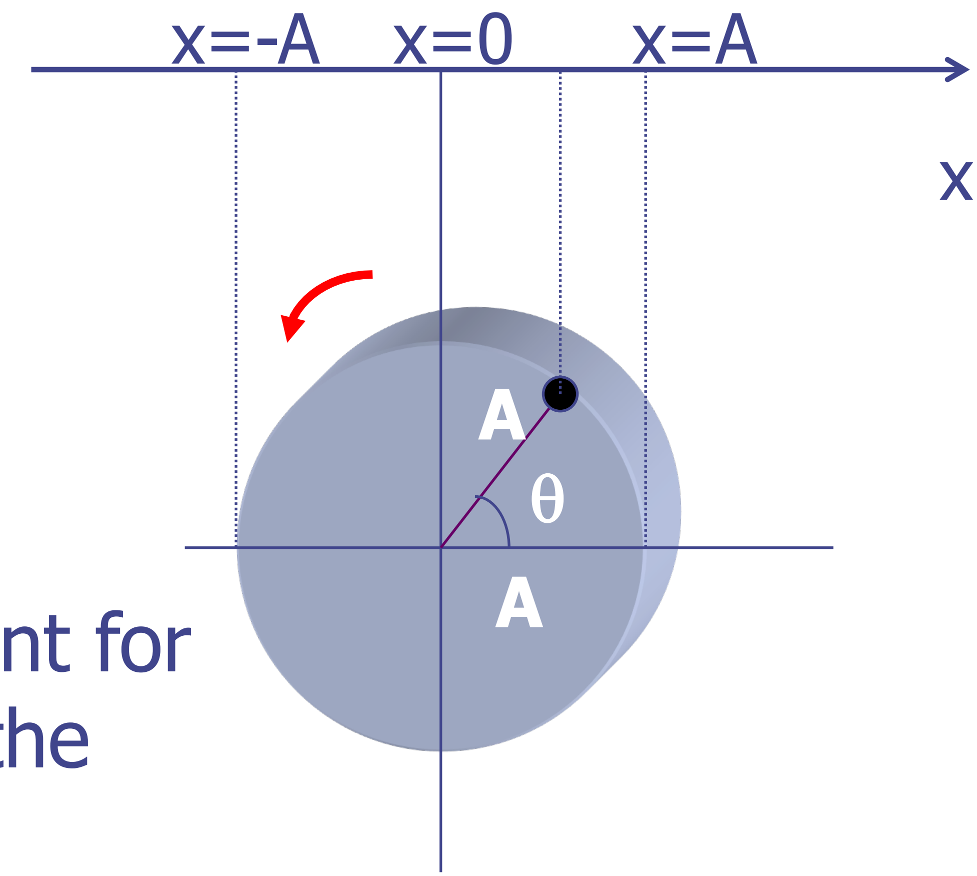
• Now, project this 2D motion onto a 1D axis

$$x = A\cos\theta$$

but $\theta = \omega t$

Therefore,

$$x = A\cos(\omega t)$$



- x is the displacement for SHM, which includes the motion of a spring
- SHM is also called sinusoidal motion
- $x_{max}=A=x_0$=amplitude of the motion (maximum)

• ω is the angular frequency (speed) in rad/s. It remains constant during the motion.

- As we saw in Chap. 4, ω and the period T are related

$$\omega = \frac{2\pi}{T}$$

- Define the *frequency*

$$f = \frac{\#\text{cycles}}{\text{sec}} = \frac{1}{T}$$

Units of 1/s = Hertz (Hz)

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Relates frequency and angular (speed) frequency

- As an example, the alternating current (AC) of electricity in the US has a frequency of 60 Hz
- Now, lets consider the velocity for SHM, again using the Reference circle

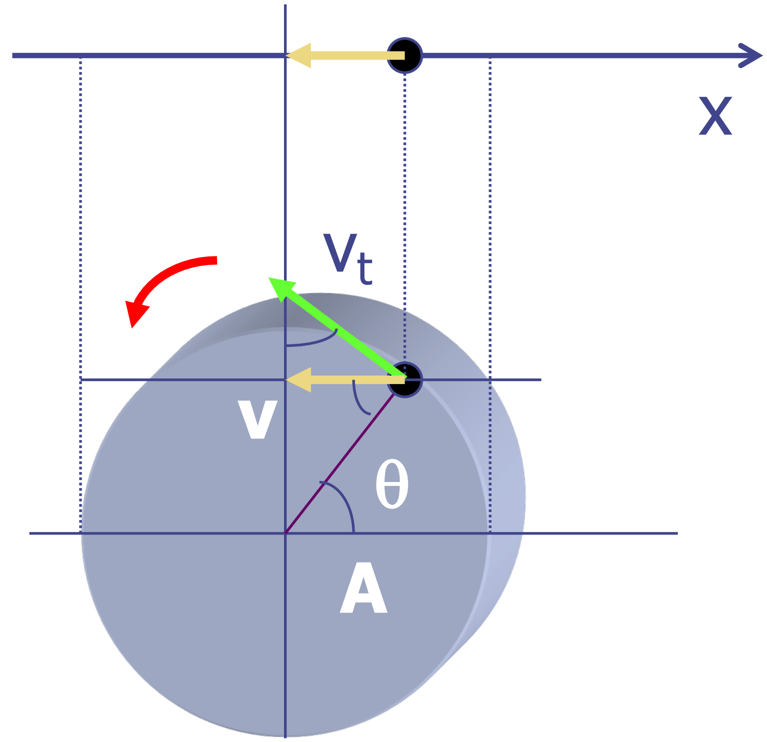
$$v = -v_t \sin\theta$$

$$v_t = r\omega = A\omega$$

$$v = -A\omega \sin(\omega t)$$

- Amplitude of the velocity is

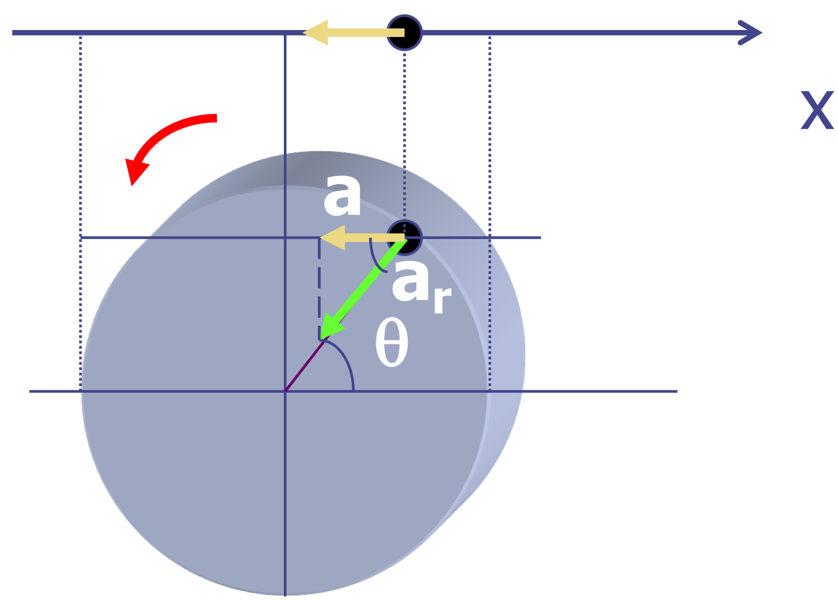
$$v_{max} = A\omega$$



- Acceleration of SHM

$$a = -a_r \cos\theta$$

$$a_r = \frac{v_t^2}{r} = \frac{r^2\omega^2}{r}$$



$$a_r = r\omega^2 = A\omega^2 \Rightarrow \boxed{a = -A\omega^2 \cos(\omega t)}$$

- The amplitude of the acceleration is

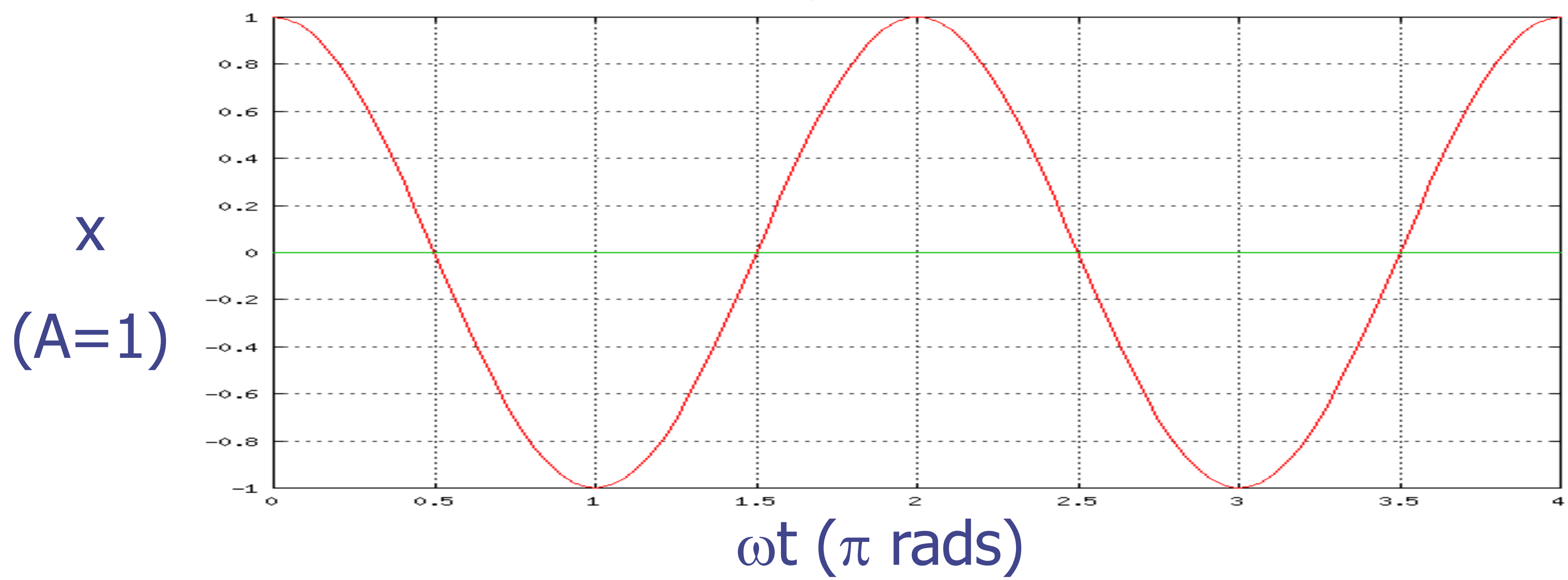
$$\boxed{a_{max} = A\omega^2}$$

- Summary of SHM

$$x = A\cos(\omega t) \qquad x_{max} = A, \ \omega t = 0, \pi, 2\pi, \ldots$$

$$v = -A\omega \sin(\omega t) \qquad v_{max} = A\omega, \ \omega t = \frac{\pi}{2}, \frac{3\pi}{2} \ldots$$

$$a = -A\omega^2 \cos(\omega t) \qquad a_{max} = A\omega^2, \ \omega t = 0, \pi, 2\pi, \ldots$$



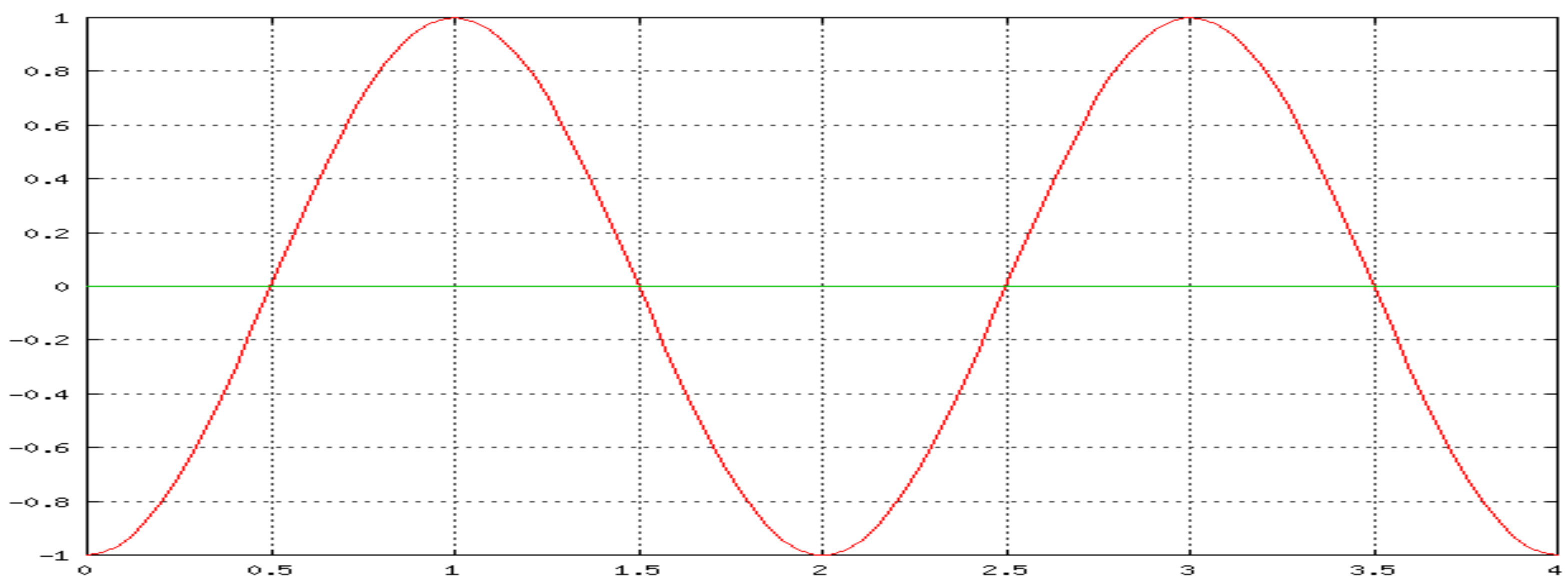
v

($A\omega=1$)

ωt (π rads)

a

($A\omega^2=1$)



Example Problem

Given an amplitude of 0.500 m and a frequency of 2.00 Hz for an object undergoing simple harmonic motion, determine (a) the displacement, (b) the velocity, and (c) the acceleration at time 0.0500 s.

Solution:

Given: A=0.500 m, f=2.00 Hz, t=0.0500 s.

$$\omega = 2\pi f = 2\pi(2.00\ \text{Hz}) = 4.00\pi\ \text{rad/s}$$

$$\omega t = (4.00\pi\ \text{rad/s})(0.0500\ \text{s})$$

$$= 0.200\pi\ \text{rad} = 0.628\ \text{rad}$$

(a) $x = A\cos(\omega t) = (0.500\ \text{m})\cos(0.628\ \text{rad}$

$$x = 0.405\,\text{m}$$

$$v = -A\omega\sin(\omega t)$$

$$v = -(0.500\,\text{m})(4\pi\ \text{rad/s})\sin(0.628\,\text{rad})$$

$$v = -3.69\,\text{m/s}$$

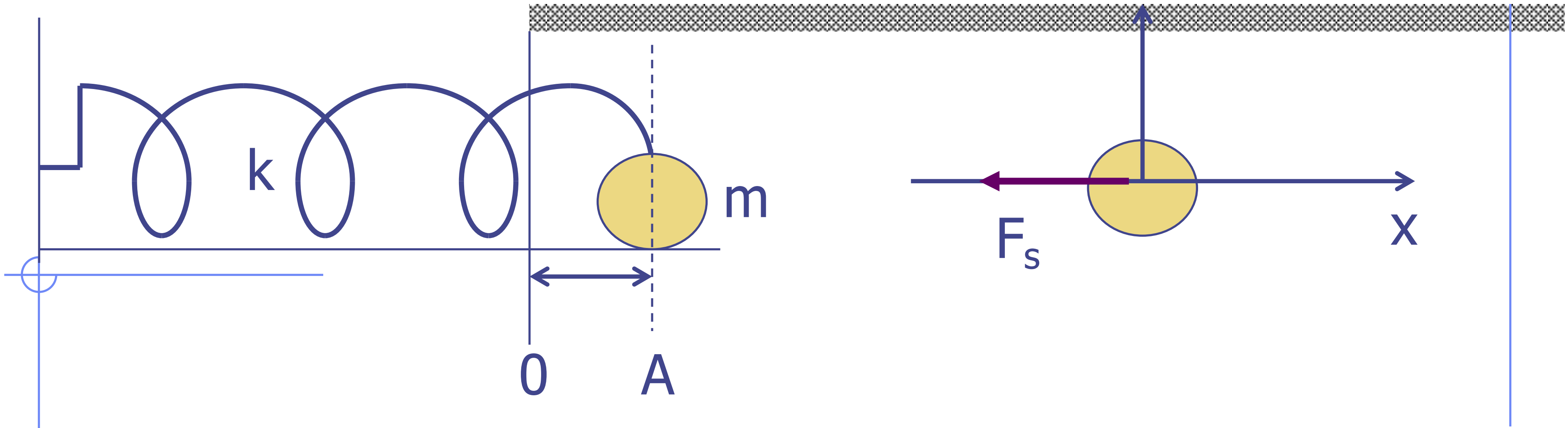
$$a = A\omega^2\cos(\omega t)$$

$$a = -(0.500\,\text{m})(4\pi\ \text{rad/s})^2\cos(0.628\,\text{rad})$$

$$a = -63.9\,\text{m/s}^2$$

Frequency of Vibration

- Apply Newton's 2nd Law to the spring-mass system (neglect friction and air resistance)



$$\sum F_x = ma_x$$ Consider x-direction only

$$F_s = -kx = ma_x$$ Substitute *x* and *a* for SHM

$$-k[A\cos(\omega t)] = m[-A\omega^2 \cos(\omega t)]$$

$$k = m\omega^2$$

$$\omega = \sqrt{\frac{k}{m}}$$

Angular frequency of vibration for a spring with spring constant *k* and attached mass *m*. Spring is assumed to be massless.

$$\omega = 2\pi f \Rightarrow \boxed{f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$$

Frequency of vibration

$$T = \frac{1}{f} = \boxed{2\pi\sqrt{\frac{m}{k}} = T}$$

Period of vibration

- This last equation can be used to determine *k* by measuring *T* and *m*

$$T^2 = 4\pi^2 \frac{m}{k} \Rightarrow \boxed{k = 4\pi^2 \frac{m}{T^2}}$$

- Note that ω (*f* or *T*) does not depend on the amplitude of the motion *A*

• Can also arrive at these equations by considering derivatives