Chap. 8 Forces Circular Motion

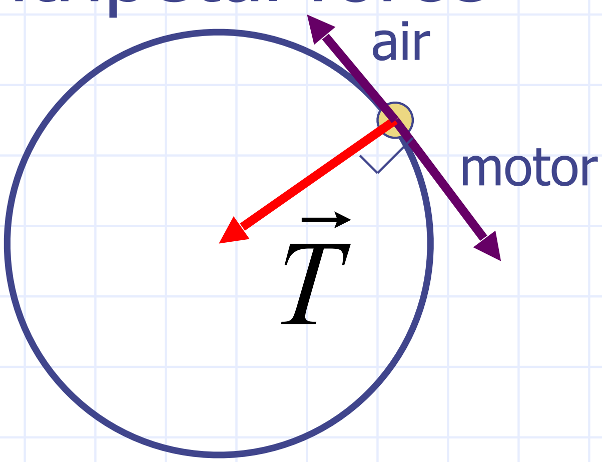
- We now look at applying Newton's 2nd law to circular (or curvilinear) motion in a plane
- We introduced the radial (or centripetal) acceleration for uniform circular motion
- $$a_r = \frac{v^2}{r}$$
- This acceleration implies a ``force"

$$\sum F_r = ma_r = \frac{mv^2}{r}$$

``centripetal force" (is not a force)

- The ``centripetal force" is the net force required to keep an object moving on a circular path

- Consider a motorized model airplane on a wire which flies in a horizontal circle, if we neglect gravity, there are only three forces, the force provided by the airplane motor which tends to cause the plane to travel in a straight line, air resistance, and the tension force in the wire, which causes the plane to travel in a circle – the tension is the ``centripetal force"

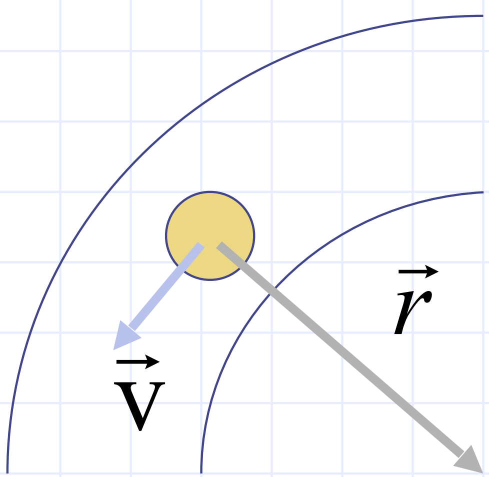


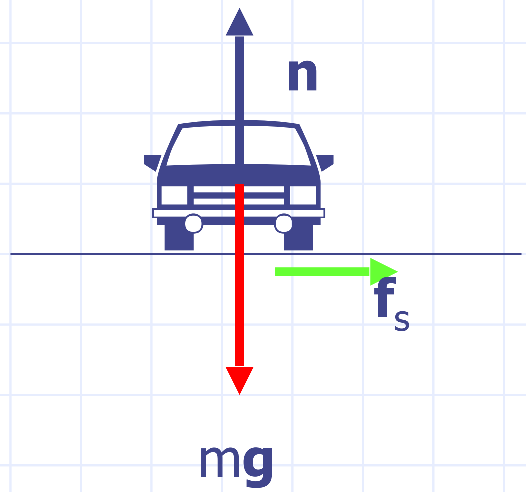
Consider forces in radial direction (positive to center)

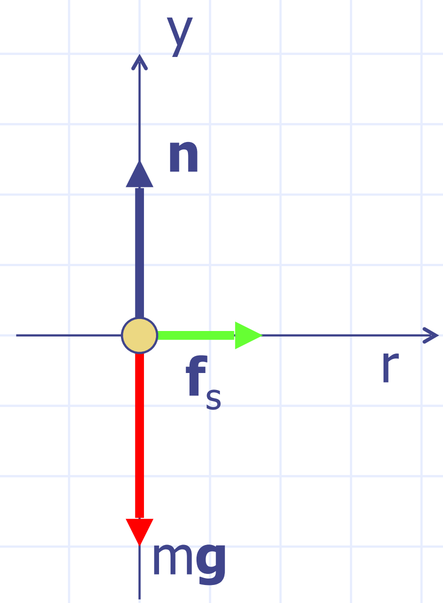
$$\sum F_r = ma_r \Rightarrow T = \frac{m\text{v}^2}{r}$$

Example (Simple)

A car travels around a curve which has a radius of 316 m. The curve is flat, not banked, and the coefficient of static friction between the tires and the road is 0.780. At what speed can the car travel around the curve without skidding?







$$\sum F_y = ma_y \qquad \sum F_r = ma_r$$

$$n - mg = 0$$

$$n = mg$$

$$f_s = \frac{m\text{v}^2}{r}$$

- Now, the car will not skid as long as f_s is less than the maximum static frictional force

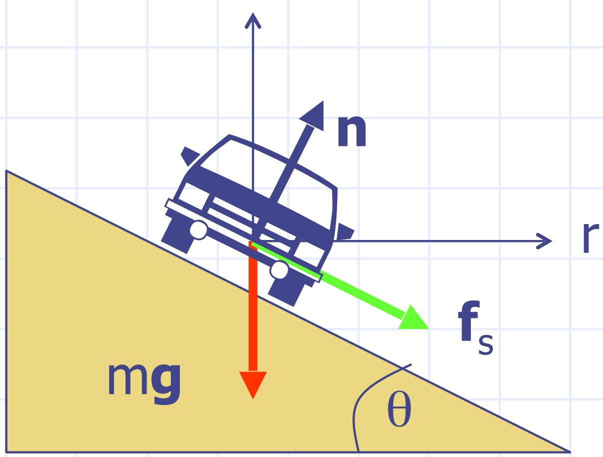
$$\frac{m\text{v}^2}{r} < f_s^{\max} = \mu_s n \Rightarrow \frac{m\text{v}^2}{r} < \mu_s mg$$

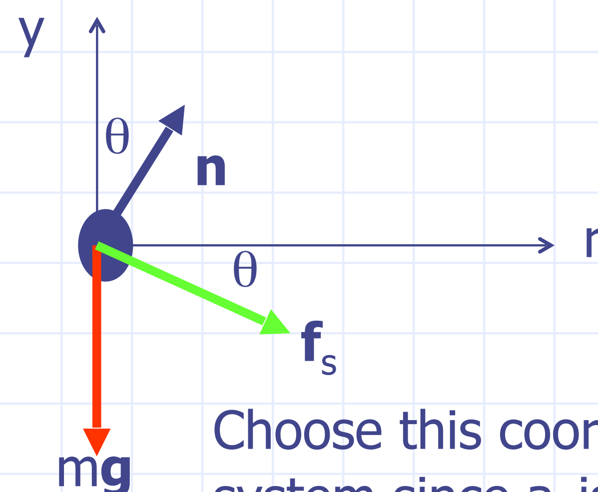
$$\text{v} = \sqrt{\mu_s g r} = \sqrt{(0.780)(9.80\,\tfrac{\text{m}}{\text{s}^2})(316\text{ m})} = 49.1\,\tfrac{\text{m}}{\text{s}}$$

$$= 49.1\,\tfrac{\text{m}}{\text{s}}\left(\frac{3600\text{ s}}{1\text{ hr}}\right)\left(\frac{1\text{ mi}}{1609\text{ m}}\right) = 110\ \tfrac{\text{mi}}{\text{hr}}$$

Example (tricky)

- To reduce skidding, use a banked curve. Consider same conditions as previous example, but for a curve banked at the angle θ





Choose this coordinate system since a_r is radial

$$\sum F_y = ma_y$$
$$n\cos\theta - f_s\sin\theta - mg = 0$$

Since acceleration is radial only

- Since we want to know at what velocity the car will skid, this corresponds to the centripetal force being equal to the maximum static frictional force

$$f_s \Rightarrow f_s^{\max} = \mu_s n$$

Substitute into previous equation

$$n\cos\theta - \mu_s n\sin\theta = mg$$

$$n(\cos\theta - \mu_s\sin\theta) = mg \Rightarrow n = \frac{mg}{\cos\theta - \mu_s\sin\theta}$$

$$\sum F_r = ma_r$$

$$n\sin\theta + f_s\cos\theta = \frac{m\mathrm{v}^2}{r}$$

$$n(\sin\theta + \mu_s\cos\theta) = \frac{m\mathrm{v}^2}{r}$$

Substitute for n and solve for v

$$\left(\frac{mg}{\cos\theta - \mu_s \sin\theta}\right)(\sin\theta + \mu_s \cos\theta) = \frac{m\mathrm{v}^2}{r}$$

$$\boxed{\mathrm{v} = \sqrt{rg\frac{(\sin\theta + \mu_s \cos\theta)}{(\cos\theta - \mu_s \sin\theta)}}}$$

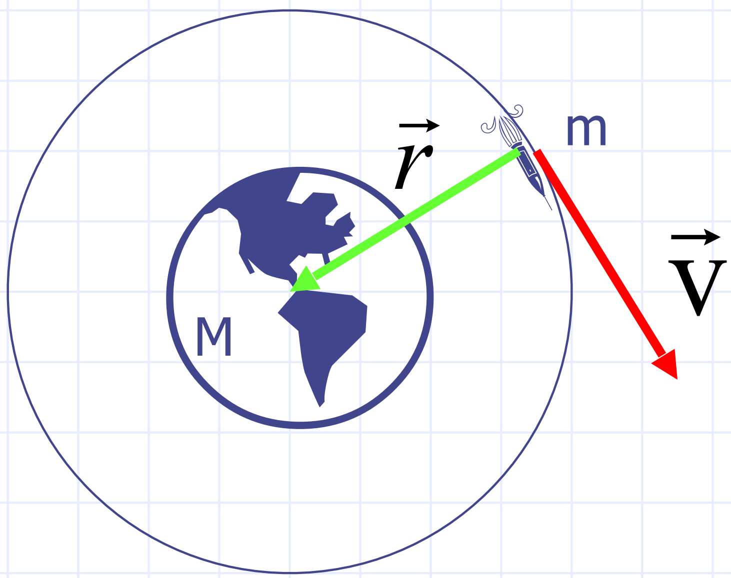
- Adopt r = 316 m and θ = 31°, and μ_s=0.780 from earlier

$$\mathrm{v} = 89.7\ \tfrac{\mathrm{m}}{\mathrm{s}} = \boxed{200\ \tfrac{\mathrm{mi}}{\mathrm{hr}}}$$

- Compare to when μ_s=0

$$\mathrm{v} = \sqrt{rg\frac{(\sin\theta)}{(\cos\theta)}} = \sqrt{rg\tan\theta} = 43.1\ \tfrac{\mathrm{m}}{\mathrm{s}} = \boxed{96.5\ \tfrac{\mathrm{mi}}{\mathrm{hr}}}$$

Orbital Motion of Satellites



- Satellites move in circular (or more generally, elliptical) orbits
- Compute their period and speed by applying Newton's 2nd Law in the radial direction

$$\sum F_r = \frac{m\mathrm{v}^2}{r}$$

Orbital speed

$$\frac{GMm}{r^2} = \frac{m\mathrm{v}^2}{r} \Rightarrow \boxed{\mathrm{v} = \sqrt{\frac{GM}{r}}}$$

$$T = \frac{2\pi r}{\mathrm{v}} = \frac{2\pi r}{\sqrt{GM/r}}$$

$$\boxed{T = \frac{2\pi r^{3/2}}{\sqrt{GM}}}$$

Orbital period

Example

Venus rotates slowly about its axis, the period being 243 days. The mass of Venus is 4.87×10^{24} kg. Determine the radius for a synchronous satellite in orbit about Venus.

Solution:

Given: $M_V = 4.87 \times 10^{24}$ kg, $T_V = 243$ days

Recognize: Synchronous means that the period of the satellite equals the period of Venus, $T_s = T_V$

Convert T_V to seconds and find r_s

$$T_V = 243\ \text{days}\left(\frac{24\ \text{hr}}{1\ \text{day}}\right)\left(\frac{60\ \text{min}}{1\ \text{hr}}\right)\left(\frac{60\ \text{sec}}{1\ \text{min}}\right) = 2.10\text{x}10^7\ \text{s}$$

$$T_S = \frac{2\pi r_s^{3/2}}{\sqrt{GM_V}} \Rightarrow r_s^{3/2} = \frac{T_s\sqrt{GM_V}}{2\pi}$$

$$r_s^{3/2} = \frac{(2.10\text{x}10^7\ \text{s})\sqrt{(6.6726\text{x}10^{-11}\ \frac{\text{Nm}^2}{\text{kg}^2})(4.87\text{x}10^{24}\ \text{kg})}}{2\pi}$$

$$= 6.025\ \text{x}\ 10^{13}\ \text{m}^{3/2} \Rightarrow \boxed{r_s = 1.54\ \text{x}\ 10^9\ \text{m}}$$

Compare this to the radius of Venus: $6.05\text{x}10^6$ m