Rotational Kinematics (4.7)

- At the end of Chapter 4, we considered rotational motion and the rotational kinematic variables (θ, ω, and α)
- Also, we considered the tangential velocity

$$v_t = \frac{2\pi r}{T} = r\omega$$

- For uniform circular motion, v_t, r, and ω are constant

- If the angular velocity changes (ω is not constant), then we have an angular acceleration α

• For some point on a disk, for example

$$v_{t,i} = r\omega_i, \quad v_{t,f} = r\omega_f$$

• From the definition of translational acceleration

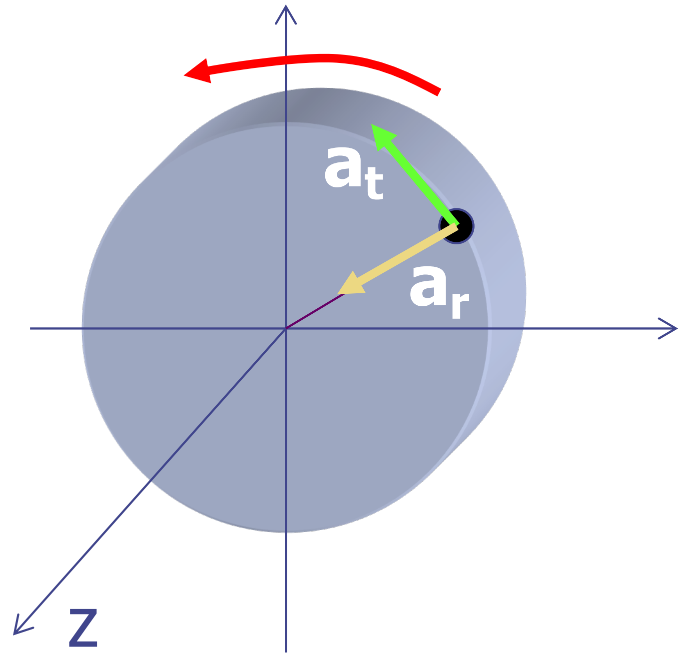
$$a = \frac{v_f - v_i}{\Delta t} = \frac{r\omega_f - r\omega_i}{\Delta t} = r\left(\frac{\omega_f - \omega_i}{\Delta t}\right)$$

$$a_t = r\alpha$$

Tangential acceleration (units of m/s^2)

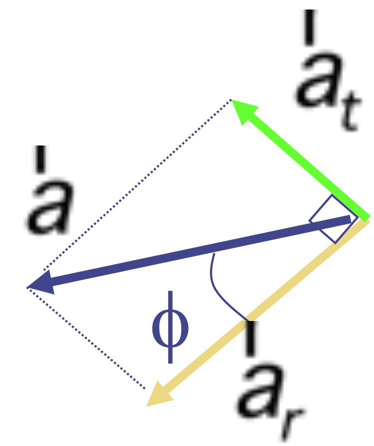
- Since the speed changes, this is *not Uniform Circular Motion*. Also, the Tangential Acceleration is different from the Centripetal Acceleration.

- Recall $a_c = \frac{v^2}{r} = \frac{v_t^2}{r} = a_r$



- We can find a total resultant acceleration **a**, since $\mathbf{a}_t$ and $\mathbf{a}_r$ are perpendicular

$$a = \sqrt{a_r^2 + a_t^2}$$
$$\phi = \tan^{-1}(a_t / a_r)$$



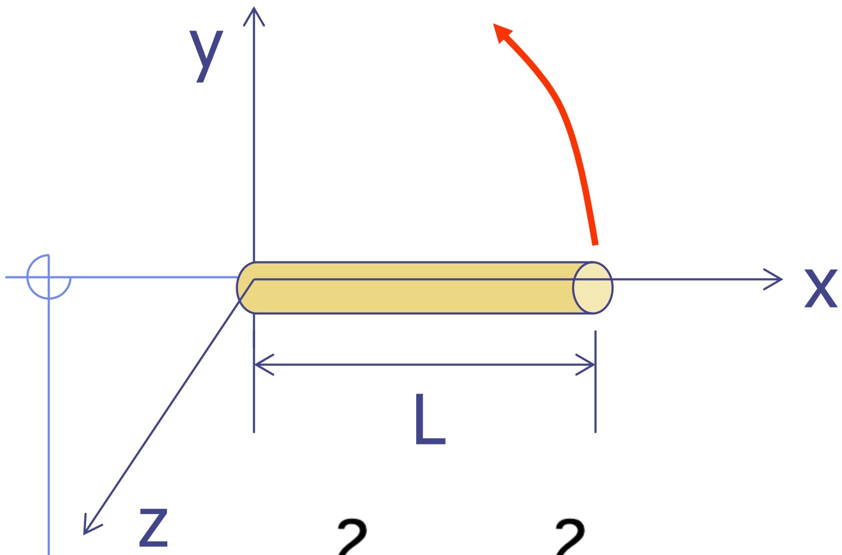
- Previously, for the case of uniform circular motion, $a_t=0$ and $a=a_c=a_r$. The acceleration vector pointed to the center of the circle.

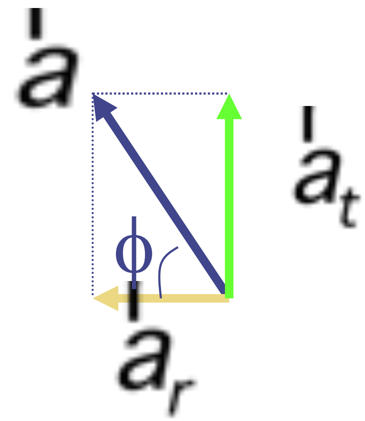
- If $a_t \neq 0$, acceleration points away from the center

Example

A thin rigid rod is rotating with a constant angular acceleration about an axis that passes perpendicularly through one of its ends. At one instant, the total acceleration vector (radial plus tangential) at the other end of the rod makes a 60.0° angle with respect to the rod and has a magnitude of 15.0 m/s^2. The rod has an angular speed of 2.00 rad/s at this instant. What is the rod's length?

Given: a = 15.0 m/s^2, ω = 2.00 rad/s (at some time)





$$a_r = \frac{v_t^2}{r} = \frac{v_t^2}{L}, \qquad a_t = r\alpha = L\alpha$$

$$v_t = r\omega = L\omega \quad \Rightarrow a_r = \frac{(L\omega)^2}{L} = L\omega^2$$

$$a_r = a\cos\phi = L\omega^2$$

Solve for L

$$L = \frac{a\cos\phi}{\omega^2} = \frac{(15.0\,\frac{\text{m}}{\text{s}^2})\cos 60.0^\circ}{(2.00\,\frac{\text{rad}}{\text{s}})^2} = \boxed{1.88\,\text{m}}$$

Equations of Rotational Kinematics

- Just as we have derived a set of equations to describe ``linear” or ``translational” kinematics, we can also obtain an analogous set of equations for rotational motion
- Consider correlation of variables

Translational		Rotational
x	displacement	θ
v	velocity	ω
a	acceleration	α
t	time	t

- Replacing each of the translational variables in the translational kinematic equations by the rotational variables, gives the set of rotational kinematic equations (for constant α)

$$\theta_f = \theta_i + \frac{1}{2}(\omega_i + \omega_f)(t_f - t_i)$$

$$\theta_f = \theta_i + \omega_i(t_f - t_i) + \tfrac{1}{2}\alpha(t_f - t_i)^2$$

$$\omega_f = \omega_i + \alpha(t_f - t_i)$$

$$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$$

- We can use these equations in the same fashion we applied the translational kinematic equations

Example Problem

A figure skater is spinning with an angular velocity of +15 rad/s. She then comes to a stop over a brief period of time. During this time, her angular displacement is +5.1 rad. Determine (a) her average angular acceleration and (b) the time during which she comes to rest.

Solution:

Given: θ_f=+5.1 rad, ω_i=+15 rad/s

Infer: θ_i=0, ω_f=0, t_i=0

Find: α, t_f ?

(a) Use last kinematic equation

$$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$$

$$0 = \omega_i^2 + 2\alpha\theta_f$$

$$\alpha = -\frac{\omega_i^2}{2\theta_f} = -\frac{(15\,\text{rad/s})^2}{2(5.1\ \text{rad})} = \boxed{-22\,\frac{\text{rad}}{\text{s}^2}}$$

(b) Use first kinematic equation

$$\theta_f = \theta_i + \tfrac{1}{2}(\omega_i + \omega_f)(t_f - t_i)$$

$$\theta_f = 0 + \tfrac{1}{2}(\omega_i + 0)(t_f - 0)$$

$$t_f = \frac{2\theta_f}{\omega_i} = \frac{2(5.1\,\text{rad})}{15\,\text{rad/s}} = \boxed{0.68\,\text{s}}$$

Or use the third kinematic equation

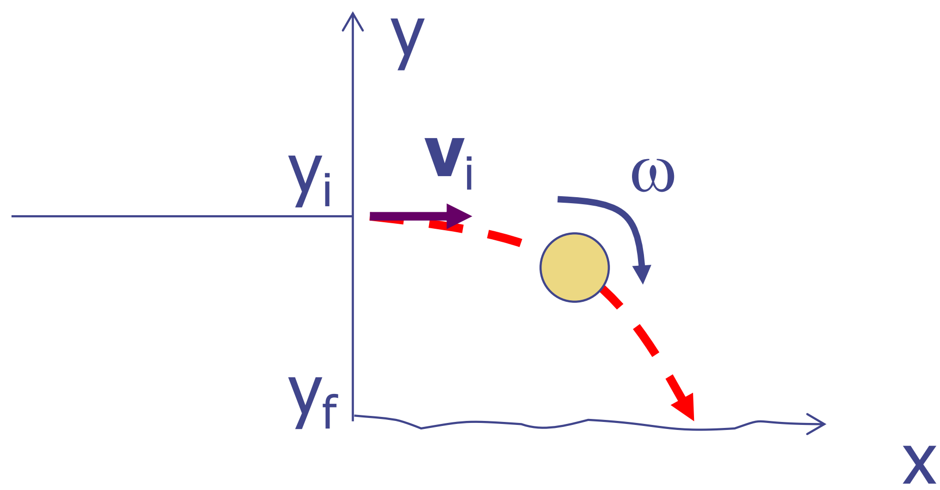
$$\omega_f = \omega_i + \alpha(t_f - t_i)$$

$$0 = \omega_i + \alpha t_f$$

$$t_f = -\frac{\omega_i}{\alpha} = -\frac{15\,\text{rad/s}}{-22\,\text{rad/s}^2} = \boxed{0.68\,\text{s}}$$

Example Problem

At the local swimming hole, a favorite trick is to run horizontally off a cliff that is 8.3 m above the water, tuck into a ``ball,” and rotate on the way down to the water. The average angular speed of rotation is 1.6 rev/s. Ignoring air resistance, determine the number of revolutions while on the way down.



Solution:

Given: $\omega_i = \omega_f = 1.6$ rev/s, $y_i = 8.3$ m

Also, $v_{yi} = 0$, $t_i = 0$, $y_f = 0$

Recognize: two kinds of motion; 2D projectile motion and rotational motion with constant angular velocity.

Method: #revolutions = $\theta = \omega t$. Therefore, need to find the time of the projectile motion, t_f.

Consider y-component of projectile motion since we have no information about the x-component.

$$y_f = y_i + v_{yi}(t_f - t_i) + \frac{1}{2}a_y(t_f - t_i)^2$$

$$0 = y_i - \tfrac{1}{2}gt_f^2 \Rightarrow y_i = \tfrac{1}{2}gt_f^2$$

$$t_f = \sqrt{\frac{2y_i}{g}} = \sqrt{\frac{2(8.3\,\mathrm{m})}{9.80\frac{\mathrm{m}}{\mathrm{s}^2}}} = 1.3\,\mathrm{s}$$

$$\theta_f = \theta_i + \tfrac{1}{2}(\omega_i + \omega_f)(t_f - t_i) = \omega t_f$$

$$\theta_f = (1.6\,\mathrm{rev/s})(1.3\,\mathrm{s}) = \boxed{2.1\,\mathrm{rev}}$$

Example Problem

- A centrifuge is a common laboratory instrument that separates components of differing densities in solutions. This is accomplished by spinning a sample around in a circle with a large angular speed. Suppose That after a centrifuge is turned off, it continues to rotate with a constant angular acceleration for 10.2 s before coming to rest. (a) if its initial angular speed was 3850 rpm, what is the magnitude of its angular acceleration? (b) How many revolutions did the centrifuge complete after being turned off?