KEY

PHYS 1211 Fall 2021 Test 1
September 21, 2021

Name ______________ **Student ID** ______________ **Score** ______

Note: This test consists of one set of conceptual questions, five problems, and a bonus problem. For the problems, you *must show all* of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last page.

Problem 1. Conceptual questions. State whether the following statements are *True* or *False*. (10 points total, no calculations required)

(a) The equation $v = \frac{1}{2}at^2$ is dimensionally correct, where v is the velocity, a the acceleration, and t the time.

False

$v = \frac{1}{2}at^2$

$[L/T] \neq \left[\frac{L}{T^2}\right][T^2] = L$

(b) For the vector sum, $\vec{R} = \vec{A}+\vec{B}$, the magnitude of the resultant vector is $|\vec{R}| = |\vec{A}|+|\vec{B}|$.

False

can only add components of a vector

$A_x + B_x = R_x$

$A_y + B_y = R_y$

(c) In a projectile motion problem near the surface of the Earth, the acceleration in the x-direction $a_x = 0$.

True

$\vec{a}_y = -g\hat{j}$

Problem 2. The position of a particle is given by the function $x = (2t^3 - 9t^2 + 12)$ m, where t is in seconds. (a) At what time(s) is $v_x = 0$ m/s? (b) What is the particle's position at this time(s)? (15 points total)

a) To find the minimum/maximum of a function, take a derivative and set to zero

$$\frac{dx}{dt} = 6t^2 - 18t = 6t(t-3) = 0 \Rightarrow \boxed{t=0 \text{ and } t=3s}$$

b) $x(t=0) = 2(0)^3 - 9(0)^2 + 12 = \boxed{12m}$

$x(t=3) = 2(3)^3 - 9(3)^2 + 12 = \boxed{-15m}$

Problem 3. $\vec{A}$ points in the negative x-direction and has a magnitude of 22.0 m. $\vec{B}$ points in the positive y-direction. (a) Find the magnitude $|\vec{B}|$ if $|\vec{R}| = |\vec{A} + \vec{B}| = 37.0$ m and (b) the direction of the resultant vector (in degrees). (c) Sketch the three vectors. (15 points total)

a) $|\vec{A}| = 22.0m$, $|\vec{B}| = ?$

$|\vec{R}| = 37.0m$

$R^2 = A^2 + B^2$

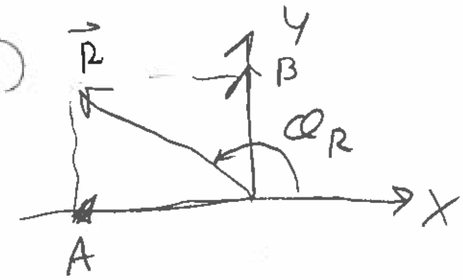
$B^2 = R^2 - A^2$

$B = \sqrt{R^2 - A^2} = \sqrt{37^2 - 22^2} = \sqrt{885}$

$= 29.7489 = \boxed{29.7m}$

b) $\theta_R = \cos^{-1}\left(\frac{-22}{37}\right) = \boxed{126.3^\circ}$

c)

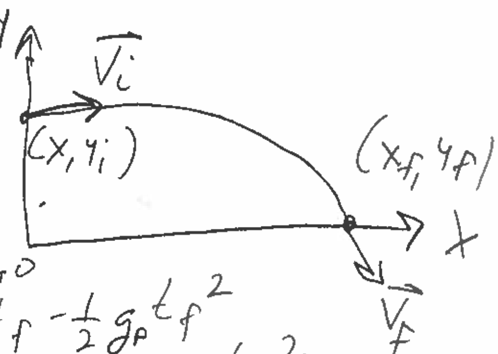


Problem 4. An astronaut on Planet X tosses a rock horizontally with a speed of 6.95 m/s. The rock falls through a vertical distance of 1.40 m and lands a horizontal distance of 8.75 m from the astronaut. What is the acceleration due to gravity on Planet X? (15 points total)

$V_{xi} = 6.95\, m/s$, $V_{yi} = 0$, $y_i = 1.4\, m$

$y_f = 0$, $x_i = 0$, $x_f = 8.75\, m$

$V_{yf} = 0$, $V_{xf} = V_{xi}$, $g_P = ?$



First find time

$x_f = x_i + V_{xi} t_f$

$x_f = V_{xi} t_f$

$t_f = \frac{x_f}{V_{xi}} = \frac{8.75\, m}{6.95\, m/s} = 1.259\, s$

Find g_P

$\cancel{y_f} = y_i + \cancel{V_{yi}} t_f - \frac{1}{2} g_P t_f^2$

$0 = y_i - \frac{1}{2} g_P t_f^2$

$g_P = \frac{2 y_i}{t_f^2} = \frac{2(1.4)}{(1.259)^2} = \boxed{1.766\, m/s^2}$

Problem 5. The radius of the Earth's very nearly circular orbit around the Sun is 1.5×10^{11} m. Find the magnitude of the Earth's (a) tangential velocity, (b) angular velocity (in rad/s), and (c) radial (or centripetal) acceleration. Assume a year of 365 days. (15 points total)

a) Period $T = 365\, days \left(\frac{24\, hrs}{1\, day}\right)\left(\frac{60\, min}{1\, hr}\right)\left(\frac{60\, sec}{1\, min}\right) = 3.1536 \times 10^7\, s$

$V_T = \frac{2\pi r}{T} = \frac{2\pi (1.5 \times 10^{11}\, m)}{3.1536 \times 10^6\, s}$

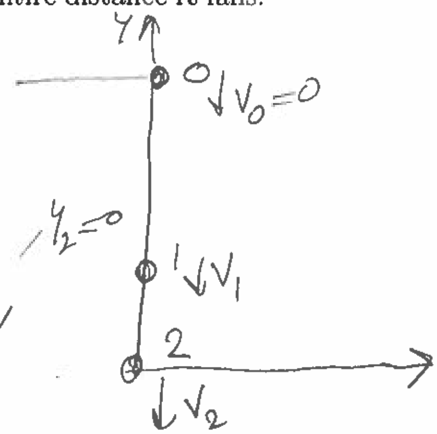
$= 29885.77 = \boxed{29.9\, km/s}$

b) $V_T = r\omega$

$\omega = \frac{V_T}{r} = \frac{2\pi r}{T r} = \frac{2\pi}{T} = \frac{2\pi\, rad}{3.1536 \times 10^7\, s} = \boxed{1.99 \times 10^{-7}\, rad/s}$

c) $a_r = \frac{V_T^2}{r} = \frac{(29885.77)^2}{1.5 \times 10^{11}\, m} = \boxed{5.95 \times 10^{-3}\, \frac{m}{s^2}}$

Problem 6. A rock is dropped from the top of a tall building (on Earth). The rock's displacement in the last second before it hits the ground is 45% of the entire distance it falls. How tall is the building? (30 points total)



$t_0 = 0$, $v_0 = 0$ (dropping subscript since 1D free fall)

$t_2 - t_1 = 1\,s$, $\frac{y_1}{y_0} = 0.45 \rightarrow y_1 = \frac{9}{20} y_0$, $y_2 = 0$

We don't know velocities at t_1 or t_2, avoid

Use equation without final velocity

$$y_f = y_i + \cancel{v_f}^{\,0}(t_f - t_i) - \frac{1}{2} g (t_f - t_i)^2$$

For ⓪ → ①

$$y_1 = y_0 - \frac{1}{2} g t_1^2$$

$$\frac{9}{20} y_0 = y_0 - \frac{1}{2} g t_1^2$$

solve for t_1

$$\frac{1}{2} g t_1^2 = \frac{11}{20} y_0$$

$$t_1 = \sqrt{\frac{22}{20} \frac{y_0}{g}}$$

For ⓪ → ②

$$y_2 = y_0 - \frac{1}{2} g t_2^2$$

$$0 = y_0 - \frac{1}{2} g t_2^2$$

Solve for t_2

$$t_2 = \sqrt{\frac{2 y_0}{g}}$$

$$t_2 - t_1 = 1 = \sqrt{\frac{2y_0}{g}} - \sqrt{\frac{22}{20}\frac{y_0}{g}} = \sqrt{\frac{2y_0}{g}}\left(1 - \sqrt{\frac{11}{20}}\right)$$

or $\sqrt{\frac{2y_0}{g}} = \left[1 - \sqrt{\frac{11}{20}}\right]^{-1} \rightarrow y_0 = \frac{g}{2}\left[1 - \sqrt{\frac{11}{20}}\right]^{-2}$

or $y_0 = \frac{9.8}{2}\left[0.2583\right]^{-2} = \boxed{73.4\,m}$

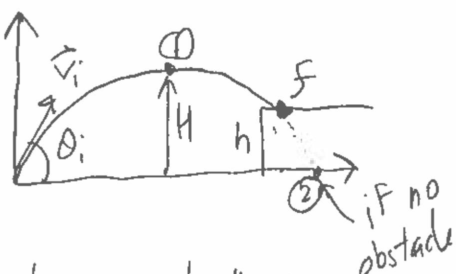
Bonus Problem. A projectile fired from $y_i = 0$ with initial speed v_i and initial velocity angle θ_i lands at height $y_f = h$ and horizontal distance x_f. Show that the total time of flight of the projectile is

$$t_f = \frac{1}{2}t_0\left(1 + \sqrt{1 - \frac{h}{H}}\right), \qquad (1)$$

$V_{xi} = V_i \cos\theta_i = V_{xf}$

$V_{yf} = V_i \sin\theta_i$

where t_0 is the time of flight when $y_f = h = 0$ and H is the maximum height of the projectile. (5 points total)



From class, get time to maximum height

$\cancel{V_{1y}}^{\,0} = V_{iy} - g t_1$

$t_1 = \frac{V_{iy}}{g} = \frac{V_i \sin\theta_i}{g}$

$t_2 = t_0 = 2t_1 = \boxed{\frac{2V_i \sin\theta_i}{g} = t_0}$

maximum height

$y_1 = H = \cancel{y_i} + \frac{1}{2}(V_{iy} + \cancel{V_{1y}})t_1$

$= \frac{1}{2}V_{iy}t_1 = \frac{V_i \sin\theta_i}{2}\frac{V_i \sin\theta_i}{g}$

$\boxed{H = \frac{V_i^2 \sin^2\theta_i}{2g}}$

Now, time to hit obstacle, t_f

$y_f = y_i + V_{iy}t_f - \frac{1}{2}g t_f^2$

$h = 0 + V_i \sin\theta_i t_f - \frac{1}{2}g t_f^2$

$t_f^2 - \left(\frac{2V_i \sin\theta_i}{g}\right)t_f + \frac{2h}{g} = 0$

of form of a quadratic equation

$az^2 + bz + c = 0, \quad z = t_f$

$a = 1, \quad b = -\frac{2V_i \sin\theta_i}{g} = -t_0$

$c = \frac{2h}{g}$ $\quad\Big|\quad z = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

Solution

$t_f = \frac{t_0}{2} \pm \frac{\sqrt{t_0^2 - 4(1)(2h/g)}}{2}$

$= \frac{t_0}{2} \pm \frac{t_0}{2}\sqrt{1 - \frac{8h}{g t_0^2}}$

$= \frac{t_0}{2}\left[1 \pm \sqrt{1 - \frac{8hg^2}{g\,4V_i^2\sin^2\theta_i}}\right]$

$= \frac{t_0}{2}\left[1 \pm \sqrt{1 - \left(\frac{2g}{V_i^2\sin^2\theta_i}\right)h}\right]$

$= \frac{t_0}{2}\left[1 \pm \sqrt{1 - \frac{h}{H}}\right]$

or $\boxed{t_f = \frac{t_0}{2}\left[1 + \sqrt{1 - \frac{h}{H}}\right]}$

$(-)$ is time on the way up