KEY

PHYS 1211 Spring 2021 Test 3
April 15, 2021

Name ____________________ **Student ID** ____________________ **Score** ________

Note: This test consists of one set of conceptual questions, five problems, and a bonus problem. For the problems, you *must show all* of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last page.

Problem 1. Conceptual questions. State whether the following statements are *True* or *False*. (10 points total, no calculations required)

(a) The center of mass of the Sun-Earth system is located in or near the Sun.

True $\quad X_{cm} = \dfrac{M_{sun}(0) + M_E R_{ES}}{M_{sun} + M_E} \sim \dfrac{M_E R_{ES}}{M_{sun}}$

(b) For an inelastic collision, the total mechanical energy is not conserved.

True $\quad$ Energy conserved for <u>elastic</u> collision

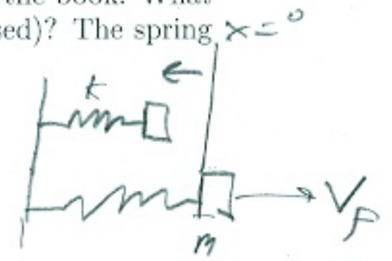
(c) The operation $\vec{A} \times \vec{A} = A^2$.

False $\quad |\vec{A} \times \vec{A}| = |\vec{A}||\vec{A}| \sin\phi = 0$, $\phi = 0$

k $m = 0.5\,kg$ $x_i = -0.04\,m$

Problem 2. A student places her 500 g physics book on a frictionless table. She pushes the book against a spring, compressing the spring by 4.0 cm, then releases the book. What is the book's speed after it slides away (with the spring finally uncompressed)? The spring constant is 1250 N/m. (15 points total)

Since no friction, energy is conserved



$$E_i = E_f, \quad E = K + U_s = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$E_i = \frac{1}{2}kx^2, \quad E_f = \frac{1}{2}mv^2$$

$x_i = -0.04\,m$

$V_i = 0$

$x_f = 0, \quad V_f = ?$

$$\cancel{\tfrac{1}{2}}kx_i^2 = \cancel{\tfrac{1}{2}}mV_f^2$$

$$V_f = \sqrt{\frac{k}{m}}\,|x_i| = \sqrt{\frac{1250}{0.5}}\,(0.04)$$

$$V_f = \boxed{2\ m/s}$$

m $V_i = 0$

Problem 3. A 0.500-kg croquet ball is initially at rest on the grass. When the ball is struck by a mallet, the average force exerted on it is 200 N. If the ball's speed after being struck is 3.00 m/s, how long was the mallet in contact with the ball? (15 points total)

V_f F_{avg}

Use Impulse-momentum theorem, since momentum of ball is not conserved

$$J = F_{avg}\Delta t = P_f - \cancel{P_i}^{\,0}$$

$V_i = 0, \ P_i = 0$

$$F_{avg}\Delta t = mV_f$$

$$\Delta t = \frac{mV_f}{F_{avg}} = \frac{(0.5)(3)}{200}$$

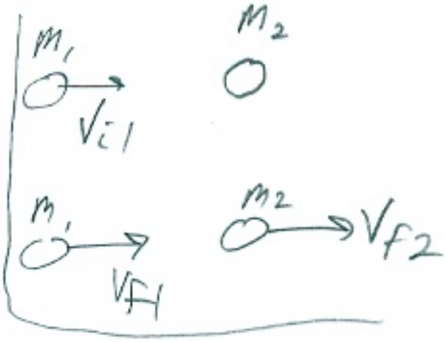
$$\Delta t = \boxed{0.0075\ s}$$

Summary $\boxed{\begin{aligned}\vec{V}_{f1} &= -1.69\times10^{6}\,\text{m/s}\ \hat{\imath} \\ \vec{V}_{f2} &= 0.308\times10^{6}\,\text{m/s}\ \hat{\imath}\end{aligned}}$

V_{i1} V_{i2}

Problem 4. A proton is traveling to the right at 2.0×10^6 m/s. It has a head-on perfectly elastic collision with a carbon atom which is initially at rest. The mass of the carbon atom is 12 times the mass of the proton. What are the speed and direction of each after the collision? Treat this as a one-dimensional problem. (15 points total)

energy conserved, momentum also conserved



$m_1 = m_p$, $m_2 = m_C = 12m_p$

$$V_{f1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)V_{i1} = \frac{m_p - 12m_p}{m_p + 12m_p}V_{i1} = \frac{-11}{13}V_{i1}$$

$$= \frac{-11}{13}(2\times10^6) = -1.69\times10^6\,\text{m/s}$$

$$V_{f2} = \left(\frac{2m_1}{m_1 + m_2}\right)V_{i1} = \frac{2m_p}{m_p + 12m_p}V_{i1} = \frac{2}{13}(2\times10^6) = 0.308\times10^6\,\text{m/s}$$

Problem 5. Starting with Newton's 2nd Law for rotation, derive the work-kinetic energy theorem for rotation:

$$\Sigma W_{\text{rotation}} = \Delta K_{\text{rotation}}, \tag{1}$$

where $K_{\text{rotation}} = \frac{1}{2}I\omega^2$. Show all steps. (15 points total)

$$\Sigma \tau_{ext} = I\alpha = I\frac{d\omega}{dt} = I\frac{d\omega}{d\theta}\frac{d\theta}{dt} = I\frac{d\omega}{d\theta}\omega$$

or

$$\Sigma \tau_{ext}\, d\theta = I\omega\, d\omega$$

$$\int \Sigma \tau_{ext}\, d\theta = \int I\omega\, d\omega = I\int_{\omega_i}^{\omega_f} \omega\, d\omega = I\left.\frac{\omega^2}{2}\right|_{\omega_i}^{\omega_f}$$

$$\Sigma W_{ext} = \frac{I}{2}(\omega_f^2 - \omega_i^2) = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$$

or $$\Sigma W_{ext}^{rot} = K_{f,rot} - K_{i,rot} = \boxed{\Delta K_{rot} = \Sigma W_{rot}}$$

$R = 0.04\,m$ $\quad 0.4\,kg$ $\quad V_i = 0$, $\omega_i = 0$ $\quad l$

Problem 6. A 8.0-cm-diameter, 400 g solid sphere is released from rest at the top of a 2.1-m-long, 25° incline. It rolls, without slipping to the bottom. (a) What is the sphere's angular velocity at the bottom of the incline? (b) What fraction of its kinetic energy is rotational? (c) If the solid sphere were replaced by a hollow sphere of the same mass and diameter, without redoing the calculations, explain whether the hollow sphere's angular velocity would be larger or smaller than that of the solid sphere from part (a). (30 points total)

Energy is conserved. Total energy is

a) $E = K_{tran} + K_{rot} + U_g = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgy$

$E_i = U_{g_i} = mgy_i = mgl\sin\theta$

$y_i = l\sin\theta$

$E_f = \frac{1}{2}mV_f^2 + \frac{1}{2}I\omega_f^2$

$= \frac{1}{2}mV_f^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{V_f}{R}\right)^2$ $\quad I = \frac{2}{5}mR^2$

$mgl\sin\theta = mV_f^2\left[\frac{1}{2} + \frac{1}{5}\right] = mV_f^2\left(\frac{7}{10}\right)$ $\quad \omega_f = \frac{V_f}{R}$

$\Rightarrow V_f = \sqrt{\frac{10}{7}gl\sin\theta} = \sqrt{\frac{10}{7}(9.8)(2.1)\sin 25°} = 3.525\,m/s$

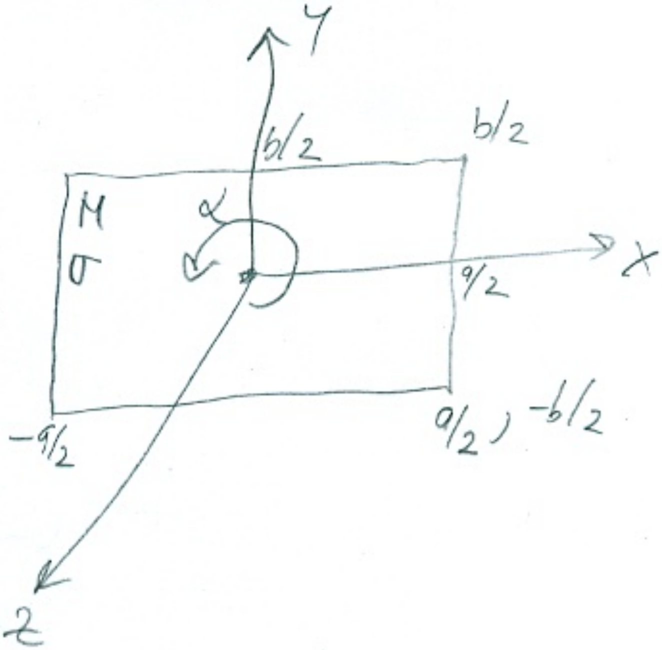
$\omega = \frac{V_f}{R} = \frac{3.525\,m/s}{0.04\,m} = \boxed{88.1\,rd/s}$

b) $\frac{K_{rot}}{K_{total}} = \frac{\frac{1}{5}mV_f^2}{\frac{7}{10}mV_f^2} = \frac{\frac{2}{10}}{\frac{7}{10}} = \boxed{\frac{2}{7}}$

c) $I_{hollow\ sphere} = \frac{2}{3}MR^2 > I_{solid\ sphere}$

since $I_{hollow} > I_{solid}$, the K_{rot} would have a larger portion of K_{total}. Then K_{trans} would be less, V_f less, then ω_f reduced

Bonus Problem. What is the torque vector needed to give a two-dimensional rectangle rotating counter-clock-wise about its center of mass an angular acceleration α? The rectangle has a mass M, uniform areal density σ, with a horizontal width a and vertical height b. Take the rectangle to be in the xy plane with its center of mass at the origin and the z-axis as the rotation axis. (5 points total)



$\tau = I\alpha$, need I about center of mass

$$I = \int r^2 dm$$

$r^2 = x^2 + y^2$ (x-y plane)

$\sigma = \frac{M}{A} = \frac{M}{ab}$

$dm = \sigma dA = \sigma dx dy$

$$I_z = \int (x^2+y^2) dm$$

$$= \int (x^2+y^2)\sigma dx\, dy$$

$$= \sigma\left[\int_{-a/2}^{a/2} x^2 dx \int_{-b/2}^{b/2} dy + \int_{-a/2}^{a/2} dx \int_{-b/2}^{b/2} y^2 dy\right]$$

$$= \frac{M}{ab}\left[\left(\frac{x^3}{3}\Big|_{-a/2}^{a/2}\right)\left(y\Big|_{-b/2}^{b/2}\right) + \left(x\Big|_{-a/2}^{a/2}\right)\left(\frac{y^3}{3}\Big|_{-b/2}^{b/2}\right)\right]$$

$$= \frac{M}{3ab}\left[\left(\frac{a^3}{8} - -\frac{a^3}{8}\right)\left(\frac{b}{2} - -\frac{b}{2}\right) + \left(\frac{a}{2} - -\frac{a}{2}\right)\left(\frac{b^3}{8} - -\frac{b^3}{8}\right)\right]$$

$$= \frac{M}{3ab}\left(\frac{a^3}{4}b + a\frac{b^3}{4}\right) = \frac{M}{12ab}ab(a^2+b^2) = \boxed{\frac{M}{12}(a^2+b^2) = I_z}$$

$$\boxed{\tau = \frac{M\alpha}{12}(a^2+b^2)}$$