KEY

PHYS 1211 Spring 2021 Test 2
March 11, 2021

Name ______________ **Student ID** ______________ **Score** ______

Note: This test consists of one set of conceptual questions, five problems, and a bonus problem. For the problems, you *must show all* **of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last page.**

Problem 1. Conceptual questions. State whether the following statements are *True* or *False*. (10 points total, no calculations required)

(a) The orbital (or tangential) speed of Jupiter is smaller than that of Earth's orbital speed because Jupiter is more massive than the Earth.

False $V_t = \sqrt{\frac{GM}{r}}$ M = mass of sun

(b) For a point on a disk with angular acceleration α, the tangential acceleration depends on the distance from the center of the disk.

True $a_t = r\alpha$

(c) An inertial frame of reference must have a constant or zero velocity with respect to another inertial frame of reference.

True

Problem 2. A 50 kg box hangs from a rope. What is the tension in the rope if (a) The box is at rest? (b) The box is moving with constant velocity $\vec{v} = 5.0\hat{j}$ m/s? and (c) The box is accelerating with $\vec{a} = 2.0\hat{j}$ m/s^2. (d) Draw free-body diagram. (15 points total)

a) $+\uparrow \Sigma F_y = ma_y$

$T - mg = ma_y$

$T = m(g + a_y)$

$a_y = 0$

$T = mg = (50\,kg)(9.8\,m/s^2)$

$= \boxed{490N}$

b) again $a_y = 0$

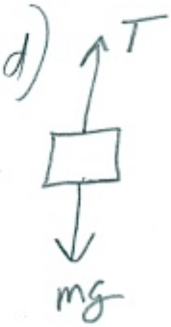
$\boxed{T = 490N}$

c) $a_y = 2\ m/s^2$

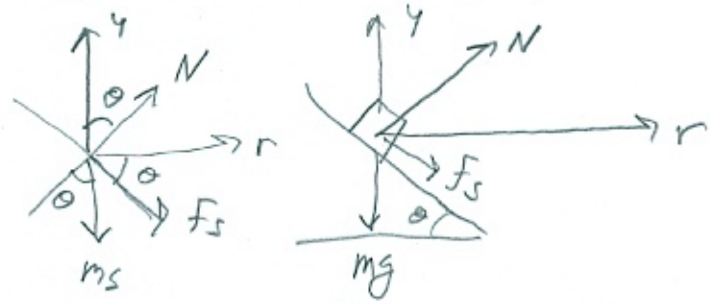
$T = 50(9.8 + 2)$

$= \boxed{590N}$

d)



Problem 3. A highway curve of radius 500 m is designed for traffic moving at a speed of 90 km/h. What is the correct banking angle? Take $\mu_s = 1.0$. Draw a free-body diagram (15 points total)



$+\uparrow \Sigma F_y = ma_y = 0$

$N\cos\theta - f_s \sin\theta - mg = 0$

$f_s \to f_s^{max} = \mu_s N$

$N\cos\theta - \mu_s N \sin\theta = mg$

$N(\cos\theta - \mu_s \sin\theta) = mg$

$N = \dfrac{mg}{\cos\theta - \mu_s \sin\theta}$ — (1)

$\overset{+}{\rightarrow} \Sigma F_r = ma_r$

$N\sin\theta + f_s\cos\theta = \dfrac{mv^2}{r}$

$N\sin\theta + \mu_s N\cos\theta = mv^2/r$

$N(\sin\theta + \mu_s\cos\theta) = \dfrac{mv^2}{r}$

substitute (1)

$\dfrac{\cancel{m}g(\sin\theta + \mu_s\cos\theta)}{(\cos\theta - \mu_s\sin\theta)} = \dfrac{\cancel{m}v^2}{r}$

$\sin\theta + \mu_s\cos\theta = \dfrac{v^2}{gr}(\cos\theta - \mu_s\sin\theta)$

$\sin\theta + \dfrac{\mu_s v^2}{gr}\sin\theta = \dfrac{v^2}{gr}\cos\theta - \mu_s\cos\theta$

$\sin\theta\left[\dfrac{\mu_s v^2}{gr} + 1\right] = \cos\theta\left[\dfrac{v^2}{gr} - \mu_s\right]$

(cont'd)

Problem 3 (Cont'd)

$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{V^2}{gr} - \mu_s}{\frac{V^2\mu_s}{gr} + 1} = \tan\theta$$

$$V = \frac{90\,km}{hr} \times \frac{1000}{1\,km} \times \frac{1\,hr}{60\,min} \times \frac{1\,min}{60\,s} = 25\,m/s$$

or $$\boxed{\theta = \tan^{-1}\left[\frac{\frac{V^2}{gr} - \mu_s}{\frac{V^2\mu_s}{gr} + 1}\right]}$$

$$= \tan^{-1}\left[\frac{\frac{25^2}{9.8(500)} - 1}{\frac{25^2(1)}{9.8(500)} + 1}\right] = \tan^{-1}\left[\frac{-0.872}{1.127}\right] = \boxed{-38^\circ}$$

Interpretation $\rightarrow$ θ can be zero or flat and no skidding

let $\mu_s \rightarrow 0$

$$\tan\theta \rightarrow \frac{V^2}{gr} \Rightarrow \theta = \tan^{-1}\left(\frac{25^2}{9.8 \cdot 500}\right) = \boxed{7.27^\circ \text{ if ice}}$$

Problem 4. A 35-cm-long vertical spring has one end fixed on the floor. Placing a 2.2 kg hunk of cheese on the spring compresses it to a length of 29 cm. What is the spring constant? (15 points total)

$l_0 = 0.35\ m, \quad l = 0.29\,m, \quad m = 2.2\ kg$

$y = l_0 - l$

$+\uparrow \Sigma F_y = m a_y$

$F_s - mg = 0$

$k(l_0 - l) = mg$

$$k = \frac{mg}{l_0 - l} = \frac{(2.2\ kg)(9.8\ m/s^2)}{0.35m - 0.29m} = \boxed{359\frac{N}{m}}$$

Problem 5. A 45 g ladybug is hovering in the air. A gust of wind exerts a force of $\vec{F} = (4.0\hat{\imath} - 6.0\hat{\jmath}) \times 10^{-2}$ N on the bug. (a) How much work is done by the wind on the ladybug as it is displaced $\Delta\vec{r} = (2.0\hat{\imath} - 2.0\hat{\jmath})$ m? (b) What is the angle between the $\vec{F}$ and $\Delta\vec{r}$? (15 points total)

a) $W = \vec{F} \cdot \Delta\vec{r} = F_x \Delta x + F_y \Delta y$

$= (4 \times 10^{-2})(2) + (-6 \times 10^{-2})(-2) = \boxed{0.2\ J}$

b) $|\vec{F}| = \sqrt{4^2 + 6^2} \times 10^{-2} = 0.072\ N$

$|\Delta\vec{r}| = \sqrt{2^2 + 2^2} = 2\sqrt{2}\ m = 2.83$

from $|\vec{A}||\vec{B}|\cos\phi = \vec{A} \cdot \vec{B}$

$|\vec{F}||\Delta\vec{r}|\cos\phi = W$

$$\cos\phi = \frac{W}{|\vec{F}||\Delta\vec{r}|} = \frac{0.2}{(0.072)(2\sqrt{2})} = 0.982$$

or $\boxed{\phi = 11.0°}$

Problem 6. A particle moving on the x-axis experiences a force given by $F_x = qx^2$, where q is a constant. (a) How much work is done on the particle as it moves from $x = 0$ to $x = d$? (b) If the particle has mass m and starts from rest at $x = 0$, find a relation for its final velocity v in terms of q, d, and m. (c) What must the dimensions of the constant q be? (30 points total)

a) $W = \int_{x_i}^{x_f} F(x)\,dx = \int_0^d q x^2 dx = q \int_0^d x^2 dx$

$= q \left[\frac{x^3}{3} \right]_0^d = \boxed{q \frac{d^3}{3}}$

b) $W = \Delta K = K_f - K_i = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \frac{1}{2} m v^2$

$\frac{1}{2} m v^2 = q \frac{d^3}{3}$

$v^2 = \frac{2}{3m} q d^3$

$\boxed{v = \pm \sqrt{\frac{2}{3} \frac{q}{m}}\, d^{3/2}}$

c) $F = q x^2$

$[N] = [\ ][m^2]$

$[\ ] = \frac{[N]}{[m^2]} = \frac{\frac{kg\,m}{s^2}}{m^2} = \boxed{\frac{kg}{m s^2}} = \boxed{\frac{N}{m^2}} = \boxed{\frac{\text{Force}}{\text{length}^2}}$

$x_i = 0, \; t_i = 0, \; \cancel{v}_i = 0$

$v_f = ?$

$a_x = ?$

Bonus Problem. A 50 kg sprinter, starting from rest, runs 50 m in 7.0 s at a constant acceleration. (a) What is the magnitude and direction of the horizontal force acting on the sprinter? (b) Explain what type of force this is. (c) What is the sprinter's power output at 2.0 s, 4.0 s, and 6.0 s. (5 points total)

a) by kinematics, Find acceleration

$$x_f = x_i + v_i t_f + \frac{1}{2} a_x t_f^2 = \frac{1}{2} a_x t_f^2$$

$$a_x = \frac{2 x_f}{t_f^2} = \frac{2(50\,m)}{(7s)^2} = 2.04\,m/s^2$$

$$\Sigma F_x = m a_x$$

$$f_s = m a_x = (50\,kg)(2.04\,m/s^2) = \boxed{102N}$$

$$\boxed{\vec{F} = 102N\,\hat{i}}$$

b) static friction forces propels the sprinter forward. The sprinter applies a force to the left, but that is an internal force (F_A). By Newton's 3rd Law $\vec{F} = -\vec{F_A}$

c) $P = \frac{dW}{dt} = \frac{d}{dt}(Fx) = F\frac{dx}{dt} = Fv$ since force is constant

$P = F(a_x t)$ since $v_f = v_i + a_x t_f$

$P = (102)(2.04)(2s) = 416\,W$

$(4s) = 832\,W$

$(6s) = 1248\,W$

instantaneous power at that time