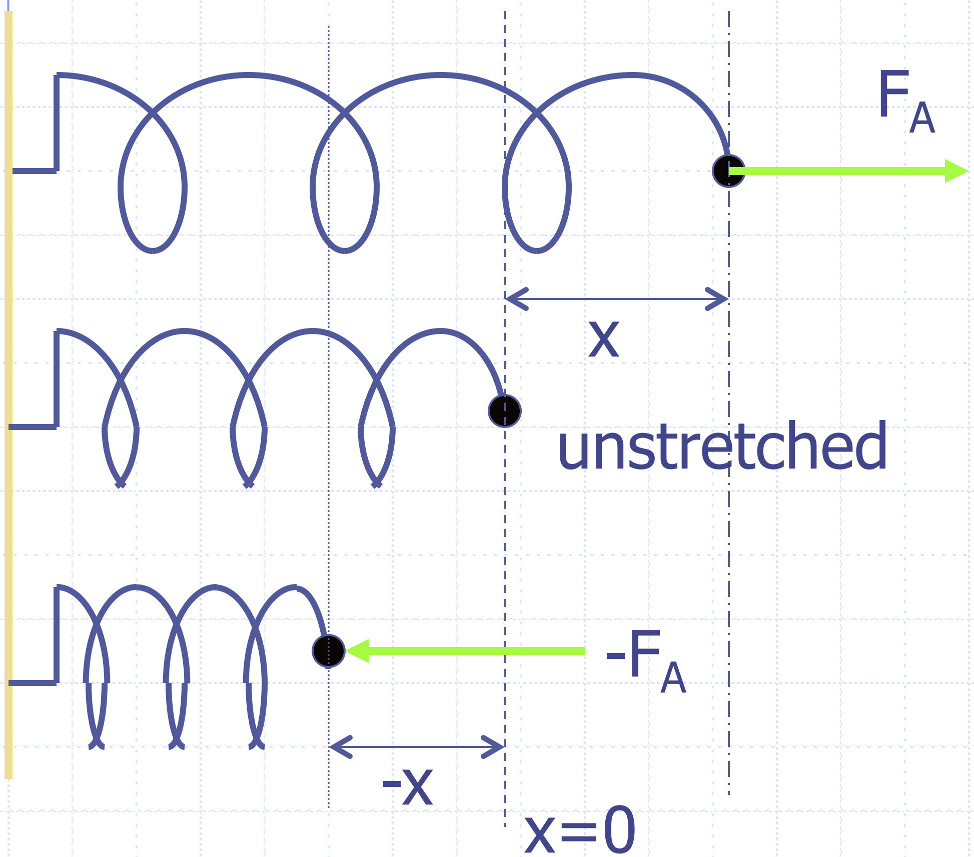
The Spring

- Consider a spring, which we apply a force F_A to either stretch it or compress it



$$F_A = kx$$

k is the spring constant, units of N/m, different for different materials, number of coils

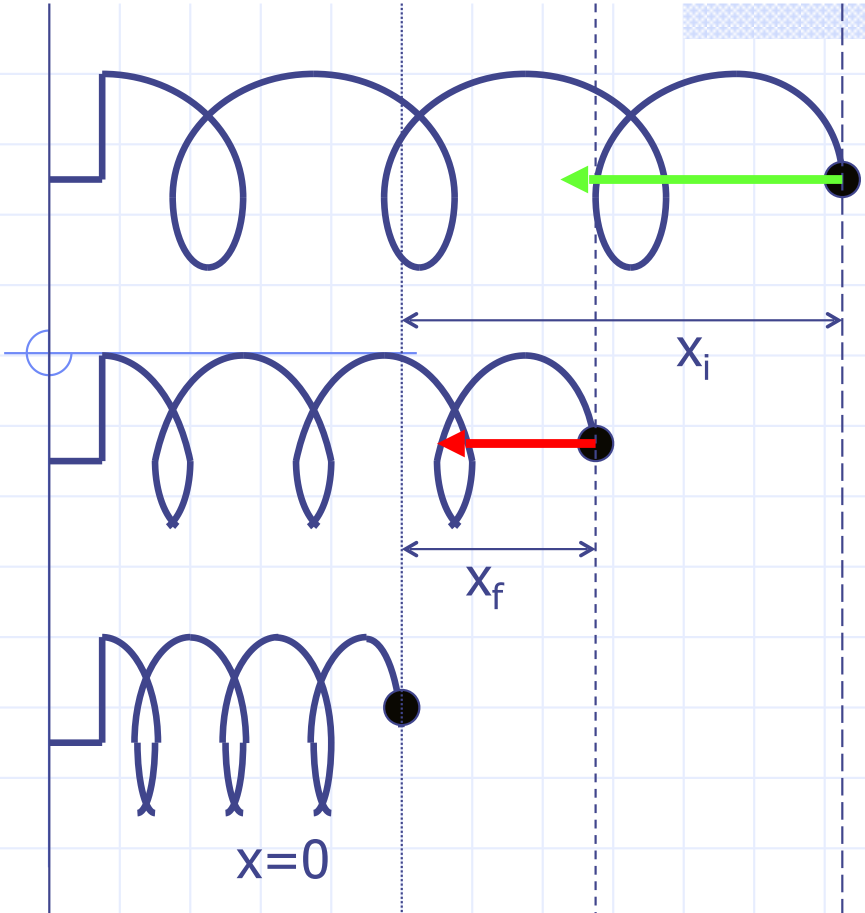
- From Newton's 3rd Law, the spring exerts a force that is equal in magnitude, but opposite in direction

$$F_s = -kx$$

Hooke's Law for the restoring force of an ideal spring.

Work done by a spring

- We know that work equals force times displacement



$$s = x_f - x_i$$

But the force is not constant

$$F_{s,i} = -kx_i,$$
$$F_{s,f} = -kx_f$$

Take the average force

$$F_{s,avg} = \frac{F_{s,i} + F_{s,f}}{2}$$

$$F_{s,avg} = -\tfrac{1}{2}k(x_f + x_i)$$

Then the work done by the spring is

$$W_s = F_{s,avg}\cos\phi\, s = F_{s,avg}\cos 0^\circ\ s$$

$$= -\tfrac{1}{2}k(x_f + x_i)(x_f - x_i) = -\tfrac{1}{2}k(x_f^2 - x_i^2)$$

$$\boxed{W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2}$$

Example problem

- If it takes 4.00 J of work to stretch a Hooke’s law spring 10.0 cm from its unstretched length, determine the extra work required to stretch it an additional 10.0 cm.

Example problem

- Using the definition of the scalar product, find the angle between the vectors:

$$\vec{A} = 3\hat{i} - 2\hat{j} \qquad \vec{B} = 4\hat{i} - 4\hat{j}$$

Power

Average power: $$P_{avg} = \frac{W}{\Delta t}$$

Units of J/s=Watt (W)

Measures the rate at which work is done

or

$$P_{avg} = \frac{F\cos\phi\, s}{\Delta t} = F\cos\phi\, \mathrm{v}_{\mathrm{avg}}$$

Instantaneous power:

$$P = \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt}$$

$$P = \frac{\vec{F}\cdot d\vec{s}}{dt} = \vec{F}\cdot\vec{\mathrm{v}}$$

W can also be replaced by the total energy E. So that power would correspond to the rate of energy transfer

Example

A car accelerates uniformly from rest to 27 m/s in 7.0 s along a level stretch of road. Ignoring friction, determine the average power required to accelerate the car if (a) the weight of the car is 1.2×10^4 N, and (b) the weight of the car is 1.6×10^4 N.

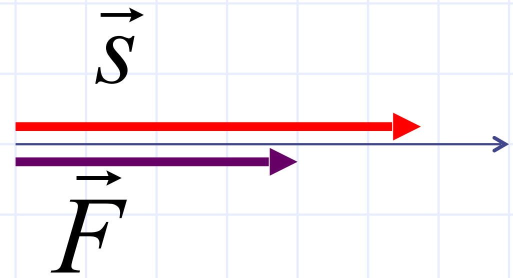
Solution:

Given: $v_i = 0$, $v_f = 27$ m/s, $\Delta t = 7.0$ s,

(a) $mg = 1.2 \times 10^4$ N, (b) $mg = 1.6 \times 10^4$ N

Method: determine the acceleration

$$P_{avg} = \frac{W}{\Delta t} = \frac{F\cos\phi\, s}{\Delta t}$$



- We don't know the displacement **s**
- The car's motor provides the force **F** to accelerate the car – **F** and **s** point in same direction

$$P_{avg} = \frac{Fs}{\Delta t} = \frac{ma_s s}{\Delta t}$$

Need a_s and s

$$v_f^2 = v_i^2 + 2a_s s \Rightarrow a_s s = \frac{(v_f^2 - v_i^2)}{2}$$

$$P_{avg} = \frac{m}{\Delta t}\left(\frac{(v_f^2 - v_i^2)}{2}\right) = \frac{m}{(7.0\ \mathrm{s})}\left(\frac{27^2 - 0}{2}\right) = 52m$$

$$P_{avg} = \frac{52mg}{g} = \frac{52(1.2\text{x}10^4)}{9.80} = \boxed{6.4\text{x}10^4 \text{ W}}$$ (a)

$$P_{avg} = \frac{52(1.6\text{x}10^4)}{9.80} = \boxed{8.5\text{x}10^4 \text{ W}}$$ (b)

Or from work-energy theorem

$$W = \Delta K = \tfrac{1}{2} m\text{v}_f^2$$

$$P_{avg} = \frac{W}{\Delta t} = \frac{m\text{v}_f^2}{2\Delta t}$$ Same as on previous slide