KEY

PHYS 1211 Spring 2021 Test 1
February 9, 2021, 9:35 am - 10:50 am

Name ________________ **Student ID** ________________ **Score** ______

Note: This test consists of one set of conceptual questions, five problems, and a bonus problem. For the problems, you *must show all* **of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last page.**

Problem 1. Conceptual questions. State whether the following statements are *True* or *False*. (10 points total, no calculations required)

(a) For uniform circular motion, the radial and centripetal acceleration vectors are perpendicular.

False parallel

(b) 1000 m/s is the same as 1 km/s.

True

(c) For vector $\vec{A}$ and scalar b, the operation $b * \vec{A}$ is valid.

True

Problem 2. Ball bearings are made by letting spherical drops of molten metal fall inside a tall tower (called a shot tower) and solidify as they fall. (a) If a bearing needs 4.0 s to solidify enough before impact, how high must the tower be? (b) What is the bearing's impact velocity? (15 points total)

1D Free-fall problem

$t_i = 0$

$t_f = 4s$

$V_i = 0$

$V_f = ?$

$y_f = 0$

$y_i = ?$

a) $y_f = y_i + V_i t_f - \frac{1}{2} g t_f^2$

$0 = y_i - \frac{1}{2} g t_f^2$

$y_i = \frac{1}{2} g t_f^2 = \frac{1}{2}(9.8 \frac{m}{s^2})(4.0s)^2$

$\boxed{y_i = 78.4m}$

b) $V_f = V_i - g t_f = -g t_f$

$V_f = -(9.8 \frac{m}{s^2})(4s)$

$V_f = -39.2 \Rightarrow \boxed{\vec{V}_f = -39.2 \hat{j}\ m/s}$

Problem 3. An old-fashioned (now hip) single-song vinyl record rotates on a turntable at 45 rpm (revolutions per minute). What are (a) the angular velocity in rad/s and (b) the period of the motion? (15 points total)

Uniform circular motion problem

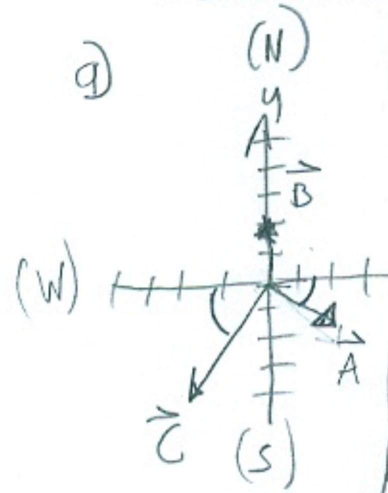
a) $\omega = 45 \frac{rev}{min} \times \frac{2\pi\ rad}{1\ rev} \times \frac{1\ min}{60s} = \boxed{4.71\ rad/s}$

b) $\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi\ rad}{4.71\ rad/s}$

$\boxed{T = 1.33s}$

Problem 4. You are given three vectors $\vec{A} = 3.0$ m, 20° south of east; $\vec{B} = 2.0$ m, north; and $\vec{C} = 5.0$ m, 70° south of west. (a) Draw all the vectors in an 2D coordinate system with their tails at the origin. (b) Write the three vectors in their component form. (c) Find the magnitude and direction of the resultant vector $\vec{D} = \vec{A} + \vec{B} + \vec{C}$. (15 points total)

a)



b) $A_x = A\cos(-20^\circ)$, $A_y = A\sin(-20^\circ)$

$= 3\cos(-20^\circ)$ $\quad = 3\sin(-20^\circ)$

$= 2.819\,m$ $\quad = -1.026\,m$

$A_x = (2.819\hat{i} - 1.026\hat{j})\,m$

$B_x = 0$, $B_y = B\sin 90^\circ = B \Rightarrow \vec{B} = 2.0\hat{j}\,m$

$C_x = C\cos(180^\circ + 70^\circ) = -1.710 \Rightarrow \vec{C} = (-1.710\hat{i} - 4.698\hat{j})\,m$

$C_y = C\sin(250^\circ) = -4.698$

c) $D_x = A_x + B_x + C_x = 1.109\,m$

$D_y = A_y + B_y + C_y = -3.724\,m$

$|\vec{D}| = \sqrt{D_x^2 + D_y^2} = \sqrt{1.109^2 + (-3.724)^2}$

$= 3.885\,m$

$\theta = \tan^{-1}\left(-\frac{3.724}{1.109}\right) = \boxed{-73.24^\circ = 286.6^\circ}$

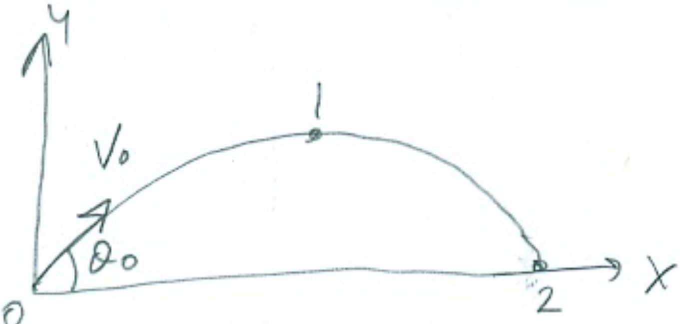
Problem 5. A particle moving in the xy-plane has velocity $\vec{v} = (2t\hat{i} + (3 - t^2)\hat{j})$ m/s, where t is in seconds. What is the particle's acceleration vector at $t = 3$ s in component form? (15 points total)

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left[2t\hat{i} + (3 - t^2)\hat{j}\right]$$

$$= 2\hat{i} + (-2t)\hat{j} = 2(\hat{i} - t\hat{j})\ m/s^2$$

$$\vec{a}\Big|_{t=3s} = 2(\hat{i} - 3\hat{j})\ \frac{m}{s^2} = \boxed{(2\hat{i} - 6\hat{j})\ m/s^2}$$

Problem 6. On the Apollo 14 mission to the moon, astronaut Alan Shepard hit a golf ball with a golf club. The free-fall acceleration on the moon is 1/6 of its value on the earth. Suppose he hit the ball with a speed of 25 m/s at an angle of 30° above the horizontal. (a) What is the time to get to the highest point of the trajectory? (b) What is the maximum height of the trajectory? (c) How far did the ball travel in the x-direction, i.e. the range? (d) Given the same initial velocity, what would be the range on the earth? (30 points total)



$g_m = \frac{g}{6}$

$V_0 = 25 m/s, \; \theta_0 = 30°$

$x_0 = 0, \; y_0 = 0, \; t_0 = 0$

$V_{0x} = V_0 \cos\theta_0 = V_{1x} = V_{2x}$

$V_{0y} = V_0 \sin\theta_0 = -V_{2y}$

a) To get highest point on trajectory

We know, $V_{1y} = 0$

$V_{1y} = V_{0y} - g_m(t_1 - t_0)$

$0 = V_{0y} - g_m t_1$

$t_1 = \frac{V_{0y}}{g_m} = \frac{V_0 \sin\theta_0}{g/6}$

$t_1 = \frac{6(25)\sin 30°}{9.8} = \boxed{7.653 s}$

b) Max. height

$y_1 = \cancel{y_0}^{0} + \frac{1}{2}(V_{0y} + \cancel{V_{1y}}^{0})t_1$

$y_1 = y_{max} = \frac{V_{0y} t_1}{2}$

$y_1 = \frac{V_0 \sin\theta_0}{2}\left(\frac{V_0 \sin\theta_0}{g/6}\right)$

$y_{max} = \frac{3V_0^2 \sin^2\theta_0}{g} = \frac{3(25)^2 \sin^2 30°}{9.8}$

$\boxed{y_{max} = 47.83 m}$

c) Range, $X_2 = V_{x2}(t_2 - t_0)$

$X_2 = V_0 \cos\theta_0 \, 2t_1 = V_0 \cos\theta_0 \, 2\frac{V_0 \sin\theta_0}{g/6}$

$X_2 = \frac{12 V_0^2 \cos\theta_0 \sin\theta_0}{g}$

$X_2 = \frac{12(25)^2 \cos 30° \sin 30°}{9.8}$

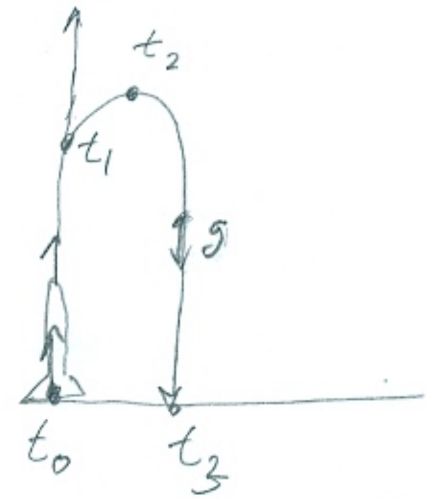
$\boxed{X_2 = 331.4 m}$

d) on Earth $g = g_m \cdot 6$

$X_2^{earth} = \frac{X_2^{moon}}{6} = \boxed{55.2 m}$

Bonus Problem. A rocket is launched straight up with constant acceleration. Four seconds after liftoff, a bolt falls off the side of the rocket. The bolt hits the ground 6.0 s later. What was the rocket's acceleration? (5 points total)

1D kinematics problem, but with an extra acceleration



$t_0 = 0$ $\quad t_1 = 4\ s$

$y_0 = 0 = y_3$ $\quad t_3 = 10\ s$

$v_0 = 0$ $\quad v_1 = ?,\ v_3 = ?,\ v_2 = 0$

- There are two accelerations for $t_0 \to t_1$ and then $t_1 \to t_2$ $\vec{a}$
- Find velocity of bolt at t_1

① $v_1 = v_0 + a(t_1 - t_0) = a t_1$

- Find height when bolt falls

$y_1 = y_0 + v_0(t_1) + \frac{1}{2} a t_1^2$

② $y_1 = \frac{1}{2} a t_1^2$

- Consider motion of bolt $t_1 \to t_3$

$y_3 = y_1 + v_1(t_3 - t_1) - \frac{1}{2} g (t_3 - t_1)^2$

let $\Delta t_{31} = t_3 - t_1$

$0 = y_1 + v_1 \Delta t_{31} - \frac{1}{2} g (\Delta t_{31})^2$

③ $y_1 = -v_1 \Delta t_{31} + \frac{1}{2} g (\Delta t_{31})^2$

→ 3 equations, 3 unknowns

→ algebra

unknowns
v_1, a
y_1, a
y_1, v_1

$\frac{②}{①}$ $\quad \frac{y_1}{v_1} = \frac{\frac{1}{2} a t_1^2}{a t_1} = \frac{t_1}{2}$

$\Rightarrow y_1 = v_1 t_1 / 2$

substitute into ③

$\frac{v_1 t_1}{2} = -v_1 \Delta t_{31} + \frac{1}{2} g (\Delta t_{31})^2$

$v_1 \left(\frac{t_1}{2} + \Delta t_{31}\right) = \frac{1}{2} g \Delta t_{31}^2$

$v_1 = \frac{\frac{1}{2} g \Delta t_{31}^2}{\frac{t_1}{2} + \Delta t_{31}} = \frac{\frac{1}{2} g (6)^2}{\frac{4}{2} + 6}$

$v_1 = 2.25 g$

From ① $\quad v_1 = a t_1$

or $a = \frac{v_1}{t_1} = \frac{2.25 g}{t_1}$

$\bar{a} = \frac{2.25(9.8)}{4} = \boxed{6.125 \frac{m}{s^2}}$