PHYS 1211, Fall 2012

Test #4, version 1

November 27, 2012

2:00 pm– 3:15 pm

1. ___________
2. ___________
3. ___________
4. ___________

Total _________

Name ____________________

ID ____________________

NOTE: This test consists of one set of conceptual question and three problems. In working the problems, you <u>must show all of the algebra and calculations</u> and your reasoning clearly to receive credit. Be sure to include units in your solutions when required.

1. Conceptual questions (10 points, no calculations required). State whether the following statements are ***True*** or ***False***.

A. For a solid uniform sphere and a thin hoop, each of mass M and radius R and rotating about their respective centers of mass, the moment of inertia of the hoop is larger than that of the sphere.

B. For a disk rotating counter-clockwise in the x-y plan through an axis about its center of mass, the direction of the angular momentum vector is in the positive z direction

C. For a mass-spring system oscillating about its equilibrium point at x=0, the magnitude of the force acting on the mass is greatest at the turning points.

2. a) Use the integral relation to **derive** the moment of inertia of a uniform rod of length L and mass M rotating about one end of the rod. b) If the rod has a mass of 2.0 kg and a length of 2.0 meters, what is its angular momentum vector if it rotates counter-clockwise about one end with an angular speed of 15 rpm? (30 points)

3. The position of a 50 g oscillating mass attached to a spring is given by $x(t) = (2.0\text{ cm})\cos(10t - \pi/4)$ where t is in s. Determine: a) the period of oscillation, b) the phase constant, c) the initial conditions (x and v at t=0), and d) the maximum speed. (30 points)

4. a) How long does it take light to travel through a 3.0-mm thick piece of window glass? b) Through what thickness of water could light travel in the same amount of time? c) What is the wavelength of light in parts a) and b)? d) If a sound wave of frequency 500 Hz traveled through the window glass and water, what would be its wavelengths in each material? Take the index of refraction for water and glass to be 1.33 and 1.5, respectively. The speed of sound in water and glass are 1480 m/s and 3960 m/s, respectively. Take the speed of light in a vacuum to be 3.0×10^8 m/s. (30 points)

Equations

$\rho = M/V$

$\mathrm{v}_{xf} = \mathrm{v}_{xi} + a_x\left(t_f - t_i\right)$ $\qquad$ $\mathrm{v}_{yf} = \mathrm{v}_{yi} + a_y\left(t_f - t_i\right)$

$x_f = x_i + \frac{1}{2}\left(\mathrm{v}_{xi} + \mathrm{v}_{xf}\right)\left(t_f - t_i\right)$ $\qquad$ $y_f = y_i + \frac{1}{2}\left(\mathrm{v}_{yi} + \mathrm{v}_{yf}\right)\left(t_f - t_i\right)$

$x_f = x_i + \mathrm{v}_{xi}\left(t_f - t_i\right) + \frac{1}{2}a_x\left(t_f - t_i\right)^2$ $\qquad$ $y_f = y_i + \mathrm{v}_{yi}\left(t_f - t_i\right) + \frac{1}{2}a_y\left(t_f - t_i\right)^2$

$\mathrm{v}_{xf}^2 = \mathrm{v}_{xi}^2 + 2a_x\left(x_f - x_i\right)$ $\qquad$ $\mathrm{v}_{yf}^2 = \mathrm{v}_{yi}^2 + 2a_y\left(y_f - y_i\right)$

$\cos\theta = x/h$ $\qquad$ $\sin\theta = y/h$ $\qquad$ $\tan\theta = y/x$ $\qquad$ $h = \sqrt{x^2 + y^2}$

$\Delta x = x_f - x_i$ $\qquad$ $\mathrm{v}_{x,avg} = \dfrac{x_f - x_i}{t_f - t_i}$ $\qquad$ $\mathbf{a}_{avg} = \dfrac{\mathbf{v}_f - \mathbf{v}_i}{t_f - t_i}$

$\mathrm{v}_x = \lim\limits_{\Delta t \to 0}\dfrac{\Delta x}{\Delta t} = \dfrac{dx}{dt}$ $\qquad$ $a_x = \lim\limits_{\Delta t \to 0}\dfrac{\Delta \mathrm{v}_x}{\Delta t} = \dfrac{d\mathrm{v}_x}{dt}$

$az^2 + bz + c = 0$ $\qquad$ $z = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Sigma\vec{F} = m\vec{a}$ $\qquad$ $\left|F_G\right| = \dfrac{Gm_1m_2}{r^2} = mg$

$f_S^{\max} = \mu_S n$ $\qquad$ $f_k = \mu_k n$

$a_r = \mathrm{v}^2/r$ $\qquad$ $T = 2\pi r/\mathrm{v} = 2\pi r^{3/2}/\sqrt{GM}$

$s = r\theta$ $\qquad$ $\omega_{avg} = \dfrac{\theta_f - \theta_i}{t_f - t_i}$ $\qquad$ $\omega = \lim\limits_{\Delta t \to 0}\dfrac{\Delta\theta}{\Delta t} = \dfrac{d\theta}{dt}$

$\alpha_{avg} = \dfrac{\omega_f - \omega_i}{t_f - t_i}$ $\qquad$ $\alpha = \lim\limits_{\Delta t \to 0}\dfrac{\Delta\omega}{\Delta t} = \dfrac{d\omega}{dt}$ $\qquad$ $T = \dfrac{2\pi}{\omega}$

$\mathrm{v}_t = r\omega$ $\qquad$ $a_t = r\alpha$ $\qquad$ $\vec{\mathrm{v}}' = \vec{\mathrm{v}} - \vec{\mathrm{v}}_o$ $\qquad$ $\vec{\mathrm{r}}' = \vec{\mathrm{r}} - \vec{\mathrm{v}}_o t$

$\theta_f = \theta_i + \frac{1}{2}\left(\omega_i + \omega_f\right)\left(t_f - t_i\right)$

$\theta_f = \theta_i + \omega_i\left(t_f - t_i\right) + \frac{1}{2}\alpha\left(t_f - t_i\right)^2$

$\omega_f = \omega_i + \alpha\left(t_f - t_i\right)$

$\omega_f^2 = \omega_i^2 + 2\alpha\left(\theta_f - \theta_i\right)$

$g = 9.80\ \mathrm{m/s^2} = 32.2\ \mathrm{ft/s^2}$ $\qquad$ $G = 6.67\mathrm{x}10^{-11}\ \mathrm{Nm^2/kg^2}$

$k_b = 1.38\mathrm{x}10^{-23}\ \mathrm{J/K}$

$$\vec{p} = m\vec{v} \qquad \vec{J} = \vec{F}_{avg}\Delta t \qquad \vec{J} = \int_{t_i}^{t_f} \vec{F}dt = \vec{p}_f - \vec{p}_i$$

$$x_{\text{cm}} = \frac{\sum m_i x_i}{\sum m_i} \qquad \vec{v}_{\text{cm}} = \frac{\sum m_i \vec{v}_i}{\sum m_i} \qquad \sum \vec{F}_{\text{ext}} = \frac{d\vec{p}}{dt}$$

$$K = \tfrac{1}{2} m v^2 \qquad W = F\cos\phi\, s = K_f - K_i$$

$$W = \int_{s_i}^{s_f} \vec{F}\cdot d\vec{s} = -\Delta U \qquad \vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z = AB\cos\phi$$

$$U_g = mgy \qquad E = K + U \qquad F_s = -kx \qquad F_x = -\frac{dU}{dx}$$

$$P_{avg} = W/\Delta t \qquad P = dW/dt \qquad W_{NC} = E_f - E_i$$

$$U_s = \tfrac{1}{2}kx^2 \qquad P = \vec{F}\cdot\vec{v} \qquad \tau = rF\sin\phi \qquad \sum \tau = I\alpha$$

$$I = \sum m_i r_i^2 = \int r^2 dm \qquad I = I_{\text{CM}} + MD^2$$

$$\lambda = M/L \qquad x_{cm} = \frac{1}{M}\int x\,dm$$

$$K_{\text{R}} = \tfrac{1}{2} I\omega^2 \qquad L = I\omega \qquad W_{\text{R}} = \tau\theta$$

$$\vec{\tau} = \vec{r}\times\vec{F} \qquad \vec{L} = \vec{r}\times\vec{p} \qquad \sum\vec{\tau} = \frac{d\vec{L}}{dt}$$

For $\vec{C} = \vec{A}\times\vec{B}$ $\qquad C = AB\sin\phi$

$$\vec{A}\times\vec{B} =$$
$$\left(A_y B_z - A_z B_y\right)\hat{i} - \left(A_x B_z - A_z B_x\right)\hat{j} + \left(A_x B_y - A_y B_x\right)\hat{k}$$

Moments of inertia about axes through the center of mass:

$$I_{rod} = \tfrac{1}{12} ML^2 \qquad I_{hollow\ sphere} = \tfrac{2}{3} MR^2 \qquad I_{solid\ sphere} = \tfrac{2}{5} MR^2$$

$$x = A\cos(\omega t + \phi_0) \qquad \omega = 2\pi f \qquad f = 1/T$$

$$v = -A\omega\sin(\omega t + \phi_0) \qquad a = -A\omega^2\cos(\omega t + \phi_0)$$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \qquad \omega = \sqrt{k/m} \qquad \omega = \sqrt{g/L}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{I/MgL} \qquad U_g = -\frac{Gm_1 m_2}{r} \qquad v = \lambda/T$$

$$v = \sqrt{\gamma k_B T/m} \qquad y(x,t) = A\sin(kx \pm \omega t + \phi_0)$$

$$k = 2\pi/\lambda \qquad n = c/v$$