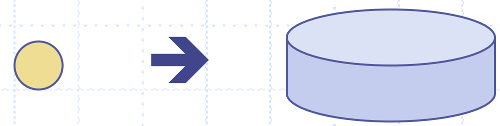
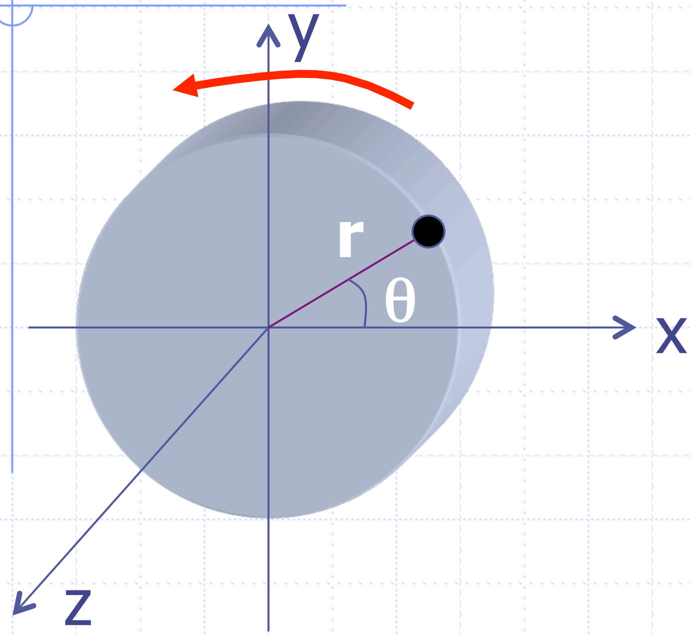
Rotational Kinematics

- Up to now, we have only considered point-particles, i.e. we have not considered their shape or size, only their mass



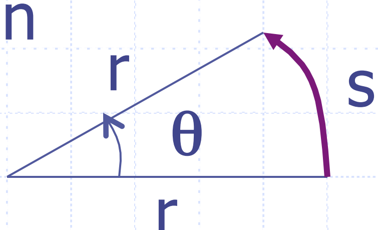
- Also, we have only considered the motion of point-particles – straight-line, free-fall, projectile motion. But real objects can also tumble, twirl, …
- This subject, rotation, is what we explore in this section and in Chapter 12.
- First, we begin by considering the concepts of circular motion

- Instead of a point-particle, consider a thin disk of radius r spinning on its axis



Axis of rotation

- This disk is a real object, it has structure
- We call these kinds of objects *Rigid Bodies*
- *Rigid Bodies* do not bend twist, or flex; for example, a billiard ball



s = arc length

$$\theta = \frac{\text{arc length}}{\text{radius}} = \frac{s}{r}$$

Units of radians (rad)

$$s = r\theta$$

- For one complete revolution $\theta = 2\pi \text{ rad}$

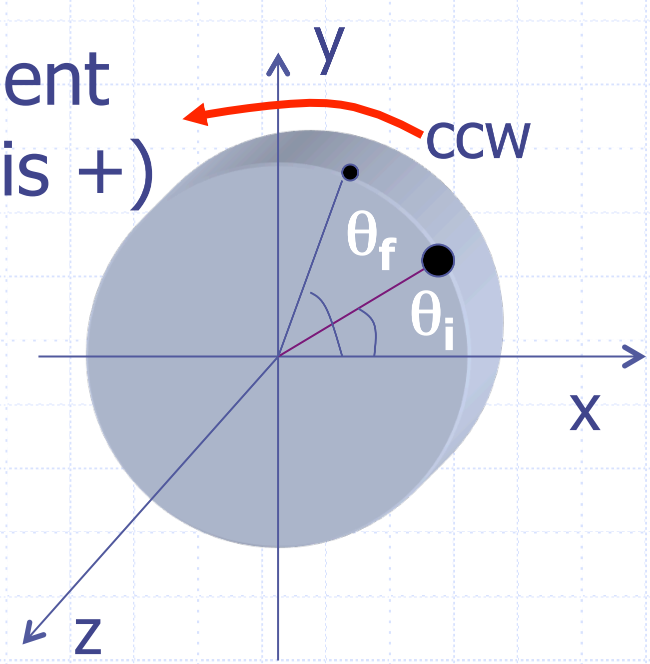
$$s = 2\pi r = \text{circumference}$$

- Conversion relation: 2π rad = 360°

- Now consider the rotation of the disk from some initial angle θ_i to a final angle θ_f during some time period t_i to t_f

$$\Delta\theta = \theta_f - \theta_i$$

Angular displacement (units of rad, ccw is +)



$$\frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t} = \omega_{avg}$$

Average angular velocity (units of rad/s)

- Similar to instantaneous velocity, we can define the *Instantaneous Angular Velocity*

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt}$$

- A change in the Angular Velocity gives

$$\frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta \omega}{\Delta t} = \alpha_{avg}$$ Average Angular Acceleration (rads/s^2)

- Analogous to *Instantaneous Angular Velocity*, the *Instantaneous Angular Acceleration* is

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt}$$

- Actually, the *Angular Velocities* and *Angular Acceleration* are magnitudes of **vector** quantities

$$\vec{\omega} \text{ and } \vec{\alpha}$$

- What is their direction?

- They point along the axis of rotation with the sign determined by the *right-hand rule*

<u>Example</u>

A fan takes 2.00 s to reach its operating angular speed of 10.0 rev/s. What is the average angular acceleration (rad/s^2)?

Solution:

Given: t_f=2.00 s, ω_f=10.0 rev/s

Recognize: t_i=0, ω_i=0, and that ω_f needs to be converted to rad/s

$$\omega_f = 10.0\,\frac{\text{rev}}{\text{s}}\left(\frac{2\pi\ \text{rads}}{1\ \text{rev}}\right) = 20.0\pi\,\frac{\text{rad}}{\text{s}}$$

$$= 62.8\,\frac{\text{rad}}{\text{s}}$$

$$\alpha_{avg} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{20.0\pi - 0}{2.00 - 0} = 10.0\pi\,\frac{\text{rad}}{\text{s}^2}$$

$$\boxed{\alpha_{avg} = 31.4\,\frac{\text{rad}}{\text{s}^2}}$$

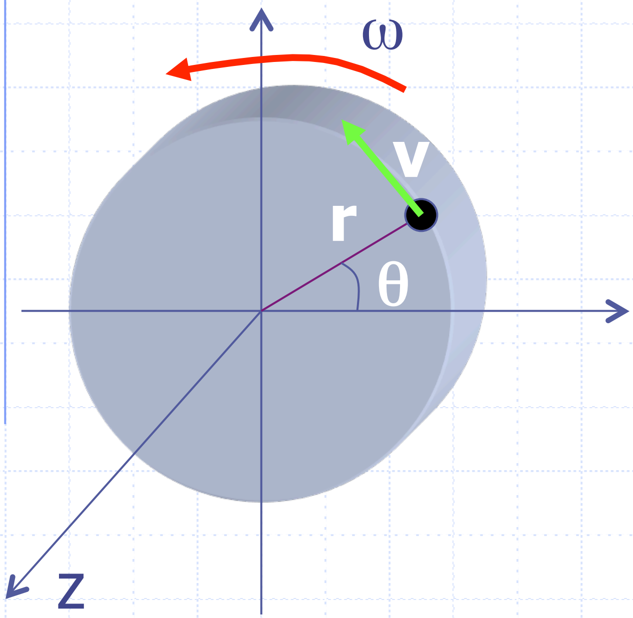
Equations of Rotational Kinematics

- Just as we have derived a set of equations to describe ``linear” or ``translational” kinematics, we can also obtain an analogous set of equations for rotational motion (section 4.7 -> later)

- Consider correlation of variables

Translational		Rotational
x	displacement	θ
v	velocity	ω
a	acceleration	α
t	time	t

Tangential Velocity



- For one complete revolution, the angular displacement is 2π rad
- From Uniform Circular Motion, we know that the time for a complete revolution is a period T
- Therefore the angular velocity (frequency) can be written

$$\omega = \frac{\Delta\theta}{\Delta t} = \boxed{\frac{2\pi}{T} = \omega} \ \text{(rad/s)}$$

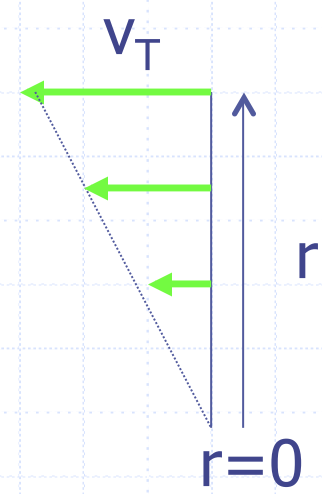
❑ Also, we know that the speed for an object in a circular path is

$$\mathbf{v} = \frac{2\pi\, r}{T} = \boxed{r\omega = \mathbf{v}_T}$$

Tangential speed (m/s)

❑ The tangential speed corresponds to the speed of a point on a rigid body, a distance r from its center, rotating at an angular speed ω

❑ Each point on the rigid body rotates at the same angular speed, but its tangential speed depends on its location r



Example Problem

- The Bohr model of the hydrogen atom pictures the electron as a tiny particle moving in a circular orbit about a stationary proton. In the lowest-energy orbit the distance from the proton to the electron is 0.529×10^{-10} m and the tangential (or linear) speed of the electron is 2.18×10^{6} m/s. (a) What is the angular speed of the electron? (b) How many orbits about the proton does it make each second?