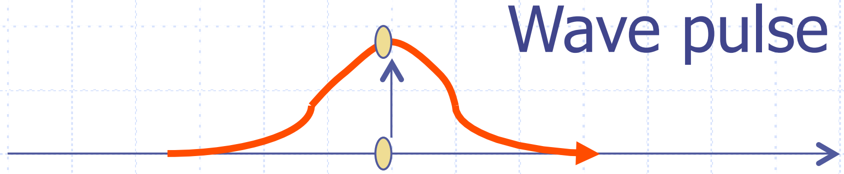
Chapter 16: Wave Motion

- We now leave our studies of mechanics and take up the second major topic of the course – wave motion (though it is similar to SHM)

- *Wave* – a traveling disturbance which carries energy from one point to another, but without the translation of mass

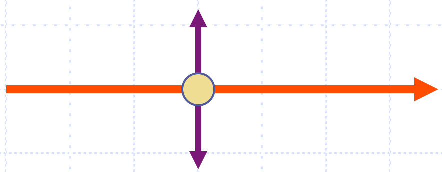


- A wave usually travels through a medium (gas, liquid, solid), except for light (an electromagnetic wave) which can propagate through a vacuum

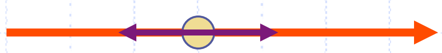
- The particles of the medium which convey the wave, can be said to oscillate about their equilibrium positions (like SHM)

- Waves can be classified according to the direction of the oscillatory displacement

- *Transverse waves* – the displacement (of the particle conveying the wave) is perpendicular to the direction of the wave; examples are a guitar string and light



- *Longitudinal waves* – the displacement is along the direction of the wave; examples are sound, the spring, and seismic waves

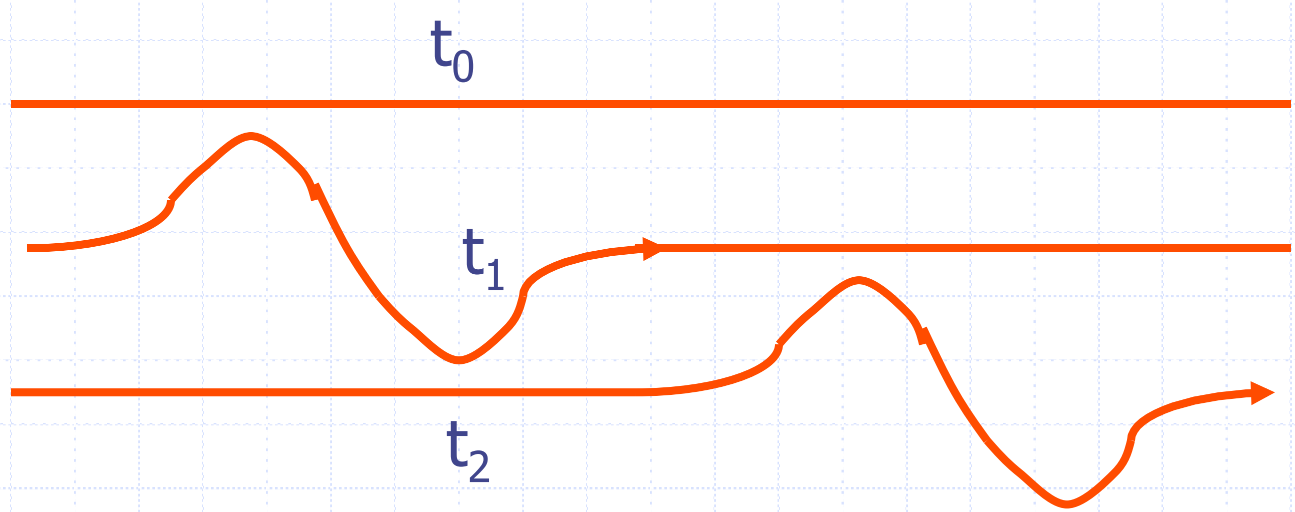


- Some waves can be a mixture of longitudinal and transverse modes – surface water wave

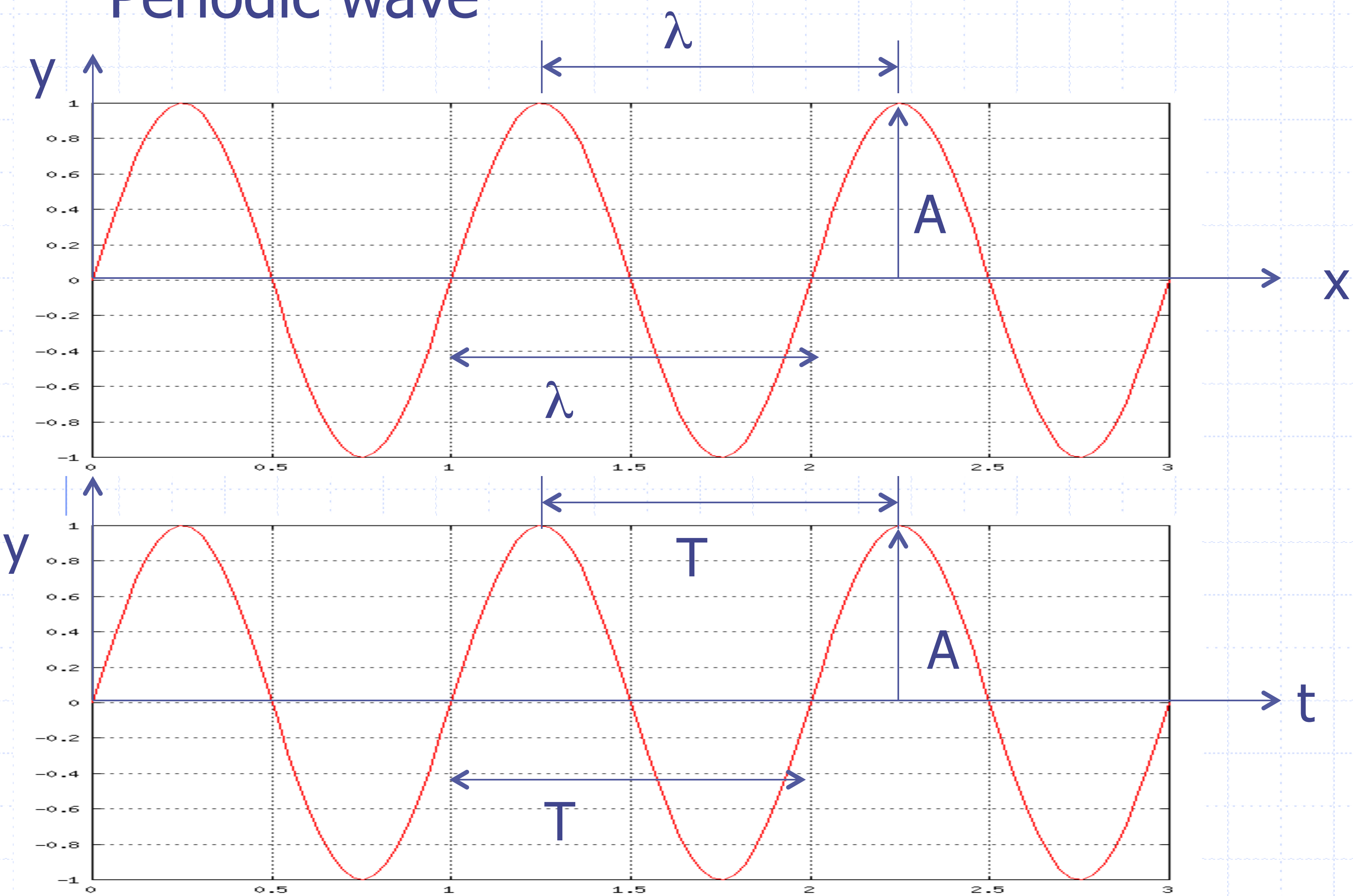
- Can also classify waves as to whether they are
  - a *Pulse* – a single cycle of disturbance
  - *Periodic* – a repeating pattern of cycles

- In both cases, each particle of the medium experiences SHM, but for the pulse they start from rest, go through one cycle, then return to rest, while for a periodic wave they experience many, many cycles

Pulse on a string



Periodic wave



Can have wave motion in both time (same as SHM) and space (x-direction for example)

- λ = wavelength, the length of one complete wave cycle; units of m

- Analogous to the Period T of a wave (for motion in time). In fact they are related

$$\lambda = vT = \frac{v}{f}$$

- Where v is the velocity of the wave. It is the velocity of a point on the wave (crest, trough, etc), not of a particle

- Since wave motion can be in space and time, we would like to have an equation for the displacement (y) as a function of space and time

- The displacement for a particle as a function of t and x (without proof):

$$y = A\sin\left(\frac{2\pi x}{\lambda} \mp 2\pi f t + \phi_0\right) = A\sin\left(kx \mp \omega t + \phi_0\right)$$

(-) positive x-direction wave motion
(+) negative x-direction wave motion
$k=2\pi/\lambda$, the angular wave number

- The quantity in parentheses is dimensionless (radians) and is called the *phase angle (φ)* of a wave

$$y = A\sin\varphi = A\cos\left(\varphi - \tfrac{\pi}{2}\right)$$

- The correspondence with Simple Harmonic Motion should be apparent

Example Problem

The speed of a transverse wave on a string is 450 m/s, while the wavelength is 0.18 m. The amplitude of the wave is 2.0 mm. How much time is required for a particle of the string to move through a distance of 1.0 km?

Solution:

Given: $v = 450$ m/s, $\lambda = 0.18$ m, $A = 2.0$ mm,
$D = 1.0$ km = travel distance of particle back and forward in the y-direction

Find: time for particle to cover distance of 1.0 km

Convert to SI units: A=2.0x10^{-3} m, D=1.0x10^{3} m

A is the distance covered in 1/4-cycle. Therefore distance covered in 1 cycle is 8.0x10^{-3} m

The period of each cycle is

$$\lambda = \mathrm{v}T$$

$$T = \frac{\lambda}{\mathrm{v}} = \frac{0.18\,\mathrm{m}}{450\,\mathrm{m/s}} = 4.0x10^{-4}\ \mathrm{s}$$

The number of cycles to cover 1.0 km is

$$\#\mathrm{cycles} = \frac{\text{total distance}}{\text{distance per cycle}} = \frac{1.0\mathrm{x}10^{3}\ \mathrm{m}}{8.0\mathrm{x}10^{-3}\ \mathrm{m}} = 1.25\mathrm{x}10^{5}$$

$$t = (\#\mathrm{cycles})T = (1.25\mathrm{x}10^{5})(4.0\mathrm{x}10^{-4}\ \mathrm{s}) = \boxed{5.0\mathrm{x}10^{1}\ \mathrm{s}}$$