PHYS 1211 Fall 2019 Test 3

November 7, 2019

Name ______________________ **Student ID** ______________________ **Score** ________

Note: This test consists of one set of conceptual questions, five problems, and a bonus problem. For the problems, you *must show all* of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last page.

Problem 1. Conceptual questions. State whether the following statements are *True* or *False*. (10 points total, no calculations required)

(a) The spring force is a non-conservative force.

F

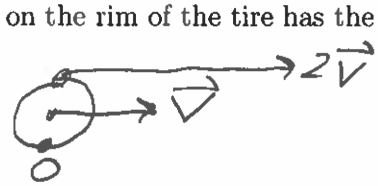
(b) For a initially stationary firecracker, the net momentum of all the fragments is zero immediately after it explodes.

T

$\vec{P}_i = 0$, Therefore $\vec{P}_f = 0$ since momentum is conserved

(c) For a tire rolling on the road, every point on the rim of the tire has the same velocity with respect to the road.

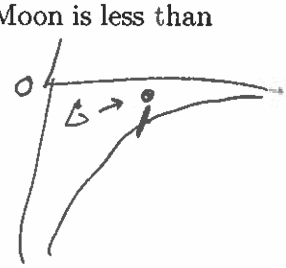
F



(d) For the case of the Moon orbiting the Earth, the total energy of the Moon is less than zero.

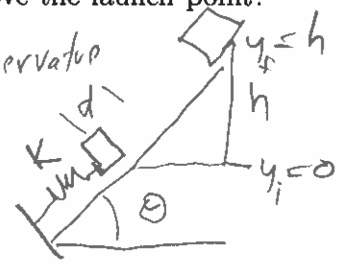
T

$$E = \frac{1}{2} m_M V^2 - \frac{G M_E m_M}{r_1} < 0$$



= 0.05kg   0.1m

Problem 2. A 50.0-g ice cube can slide up and down a frictionless 30° slope. At the bottom, a spring with spring constant 25 N/m is compressed 10.0 cm and is used to launch the ice cube up the slope. What is the maximum height for the ice cube above the launch point? (15 points total)



Use conservation of energy, no non-conservative forces

$$E = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2$$

$$E_i = \frac{1}{2}kd^2 \qquad E_f = mgy_f = mgh$$

$$v_i = 0, \; v_f = 0$$

$$\frac{1}{2}kd^2 = mgh$$

$$h = \frac{kd^2}{2mg} = \frac{(25\,N/m)(0.1m)^2}{2(0.05\,kg)(9.8\,m/s^2)} = \boxed{0.255m}$$

Problem 3. A particle moving along the y-axis is in a system with potential energy $U(y) = 4y^3$ J, where y is in m. What is the y-component of the force on the particle at $y = 2$ m? (15 points total)

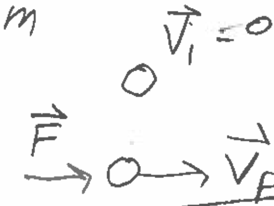
$$F_y = -\frac{dU(y)}{dy} = -\frac{d}{dy}(4y^3) = -(4)(3)y^2$$

$$F_y = -12y^2 = -12(2)^2 = \boxed{-48N}$$

m $\qquad V_i = 0, \; P_i = 0$

Problem 4. A 0.505-kg croquet ball is initially at rest on the grass. When the ball is struck by a mallet, the average force exerted on it is 242 N. If the ball's speed after being struck is 3.00 m/s, how long was the mallet in contact with the ball? (15 points total)

V_f Use impulse momentum theorem



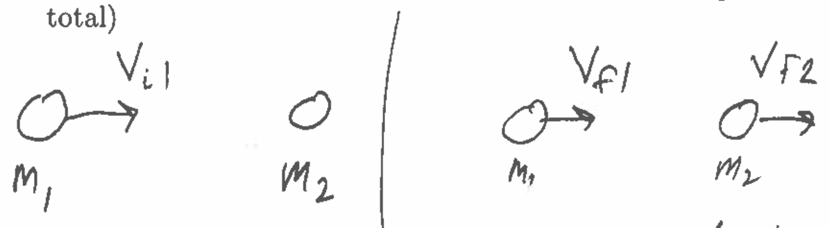
$$\vec{J} = \vec{F}\Delta t = \Delta \vec{p}$$

$$F\Delta t = P_f - P_i = P_f = mV_f$$

$$\Delta t = \frac{mV_f}{F} = \frac{(0.505\,kg)(3.0\,m/s)}{242\,N} = \boxed{6.26\times10^{-3}\,s}$$

V_{i1}

Problem 5. A proton is traveling to the right at 2.0×10^7 m/s. It has a head-on perfectly elastic collision with a carbon atom in one-dimension. The carbon atom has a mass 12 times that of the proton. What are the velocities of each particle after the collision? (15 points total)



$m_1 = m_0, \; m_2 = 12m_0$

energy and momentum are conserved, Use equations from equation sheet

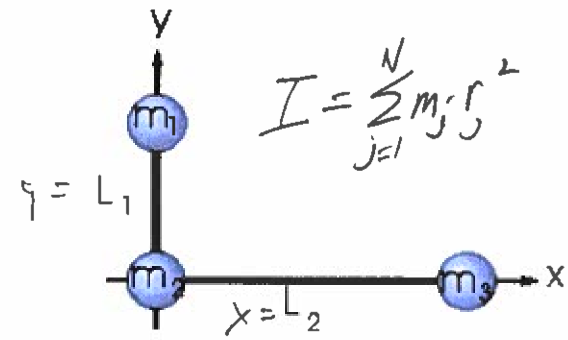
$$V_{f1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)V_{i1} = \left(\frac{m_0 - 12m_0}{m_0 + 12m_0}\right)V_{i1} = \frac{-11}{13}V_{i1}$$

$$= -\frac{11}{13}(2\times10^7\,m/s) = \boxed{-1.69\times10^7\,m/s}$$

$$V_{f2} = \frac{2m_1}{m_1 + m_2}V_{i1} = \frac{2m_0}{m_0 + 12m_0}V_{i1} = \frac{2}{13}V_{i1} = \frac{2}{13}(2\times10^7\,m/s)$$

$$= \boxed{3.08\times10^6\,m/s}$$

Problem 6. Given the system in the figure composed of three masses (m_1=1.0 kg, m_2=2.0 kg, m_3=3.0 kg) separated by the lengths L_1=1.0 m and L_2=2.0 m, determine the following (a) the moment of inertia about the x-axis, (b) the moment of inertia about the y-axis, and (c) the moment of inertia about the z-axis. (d) If a force $F = \langle 0, 0, -2.0 \rangle$ N acts on m_3, what is the resulting torque and angular acceleration? (e) In Part (d), about which axis does the rotation occur? (30 points total)



$$I = \sum_{j=1}^{N} m_j r_j^2$$

a) $I_x = m_1 y^2 = m_1 L_1^2$

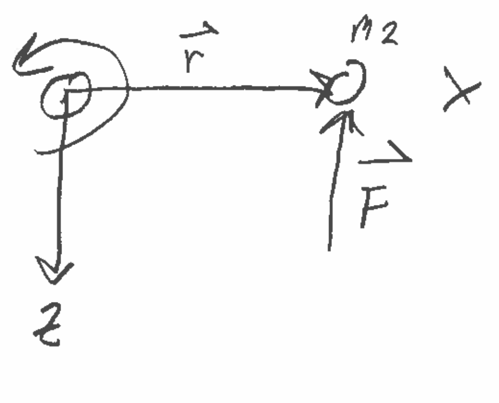
$= (1.0\,kg)(1.0\,m)^2 = \boxed{1\ kg\,m^2}$

b) $I_y = m_3 X^2 = m L_2^2$

$= (3.0\,kg)(2.0\,m)^2 = \boxed{12\ kg\,m^2}$

c) $I_z = m_1 L_1^2 + m_3 L_2^2 = I_x + I_y = \boxed{13\ kg\,m^2}$

d)



$\tau = |\vec{F}||\vec{F}| = L_2 F_z$

$= (2.0\,m)(2\,N) = \boxed{4\,Nm\ ccw}$

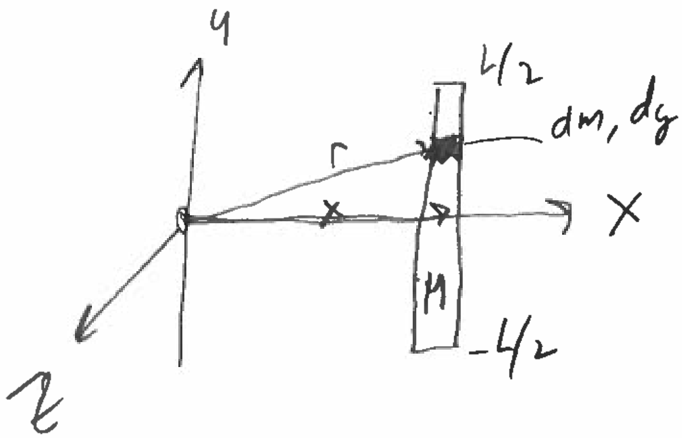
e) $\boxed{y\text{-axis}}$

$\Sigma \tau = I\alpha$

$\tau = I_y \alpha$

$\alpha = \dfrac{\tau}{I_y} = \dfrac{4\,Nm}{12\,kg\,m^2} = \boxed{\dfrac{1}{3}\ rad/s^2}$ ccw

Bonus Problem. A uniform rod of mass M and length L is placed with its center of mass at position x, but with the rod in the vertical position parallel to the y-axis. What is the moment of inertia for the rod about the z-axis? (5 points total)



$\lambda = M/L$, $M = \lambda L$

$dm = \lambda dy$

$r^2 = x^2 + y^2$

$$I_z = \int r^2 dm = \int (x^2+y^2)\lambda dy = x^2 \int_{-L/2}^{L/2} \lambda dy + \lambda \int_{-L/2}^{L/2} y^2 dy$$

$$= x^2 \lambda y \Big|_{-L/2}^{L/2} + \lambda \frac{y^3}{3}\Big|_{-L/2}^{L/2}$$

$$= x^2 \lambda \left[\frac{L}{2} - \left(-\frac{L}{2}\right)\right] + \frac{\lambda}{3}\left[\left(\frac{L}{2}\right)^3 - \left(-\frac{L}{2}\right)^3\right]$$

$$= x^2 \lambda L + \frac{\lambda}{3}\left[\frac{L^3}{8} + \frac{L^3}{8}\right] = Mx^2 + \frac{M/L}{3}\left[\frac{L^3}{4}\right]$$

$$\boxed{I_z = MX^2 + \frac{ML^2}{12}}$$

in current configuration $x \to r$ for rotation about z

Could use parallel axis thm

$I_z = MD^2 + I_{cm}$

$I_z = MX^2 + \frac{M}{12}L^2$