Chapter 12: Angular Momentum

Static Equilibrium

- In Chap. 4 we studied the equilibrium of point-objects (mass m) with the application of Newton's Laws

$$\sum F_x = 0, \ \sum F_y = 0$$

- Therefore, no linear (translational) acceleration, **a**=0

- For rigid bodies (non-point-like objects), we can apply another condition which describes the lack of rotational motion

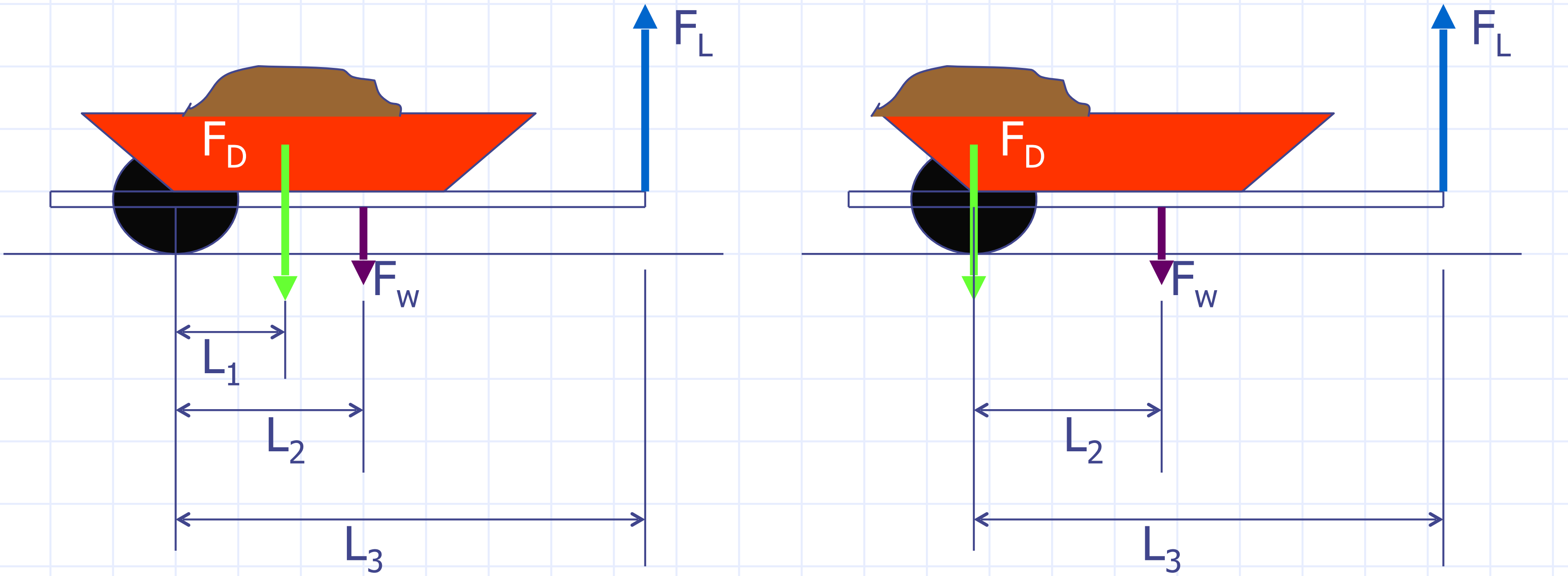
$$\sum \vec{\tau} = 0$$

- If the net of all the applied torques is zero, we have no rotational (angular) acceleration, $\alpha=0$ (don't need to know moment of inertia)

- We can now use these three relations to solve problems for rigid bodies in equilibrium (**a**=0, $\alpha=0$)

Example Problem

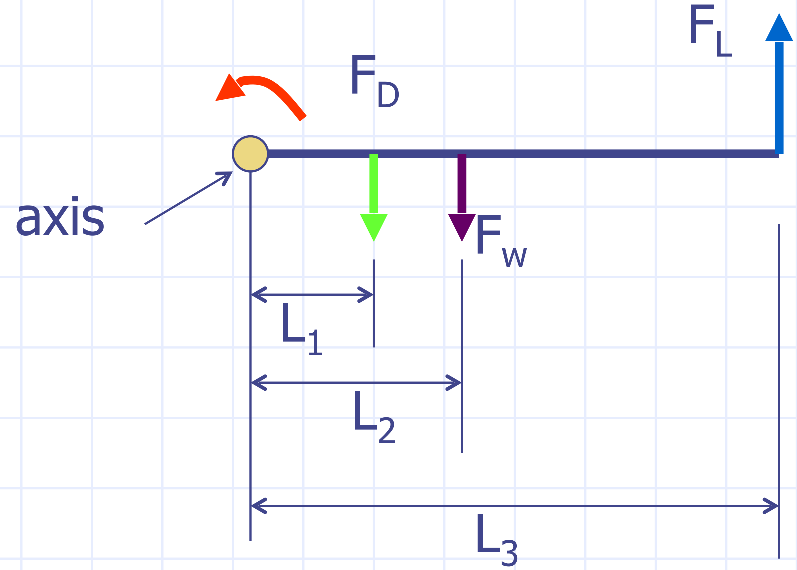
The wheels, axle, and handles of a wheelbarrow weigh 60.0 N. The load chamber and its contents weigh 525 N. It is well known that the wheel-barrow is much easier to use if the center of gravity of the load is placed directly over the axle. Verify this fact by calculating the vertical lifting load required to support the wheelbarrow for the two situations shown.



L_1 = 0.400 m, L_2 = 0.700 m, L_3 = 1.300 m

- First, draw a FBD labeling forces and lengths from the axis of rotation

  Choose a direction for the rotation, CCW being positive is the convention



a) $$\sum \tau = 0$$

$$\tau_D + \tau_W + \tau_L = 0$$

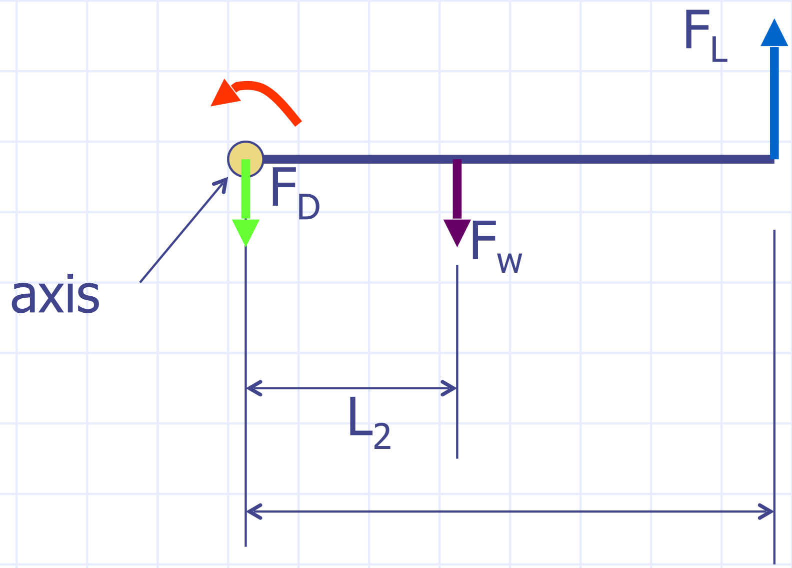
$$-F_D L_1 - F_W L_2 + F_L L_3 = 0$$

$$F_L = \frac{F_D L_1 + F_W L_2}{L_3}$$

$$F_L = \frac{(525\text{ N})(0.400\text{ m}) + (60.0\text{ N})(0.700\text{ m})}{1.300\text{ m}}$$

$$F_L = \boxed{194\text{ N}}$$

- Apply to case with load over wheel



- Torque due to dirt is zero, since lever arm is zero

b) $$\sum \tau = 0$$

$$\tau_D + \tau_W + \tau_L = 0$$

$$-F_D L_1 - F_W L_2 + F_L L_3 = 0$$

$$F_L = \frac{F_D L_1 + F_W L_2}{L_3}$$

$$F_L = \frac{(525\ \text{N})(0\ \text{m}) + (60.0\ \text{N})(0.700\ \text{m})}{1.300\ \text{m}}$$

$$F_L = \boxed{32.3\ \text{N}}$$

- Who? What is carrying the balance of the load?

- Consider sum of forces in y-direction

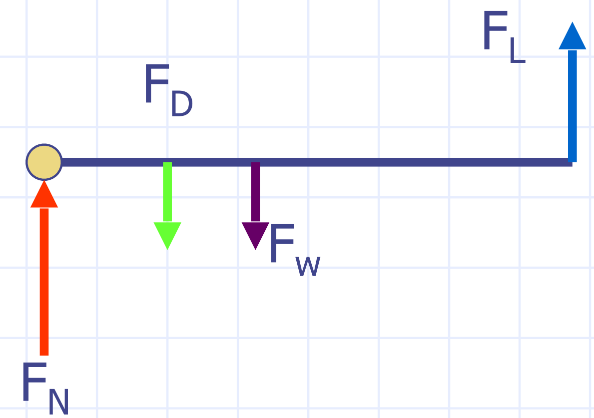
$$\sum F_y = 0$$

$$F_L - F_D - F_W + F_N = 0$$

$$F_N = F_D + F_W - F_L$$

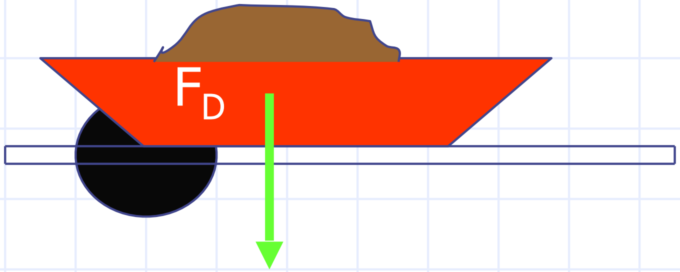
a) $$F_N = 525 + 60 - 194 = \boxed{391\,\text{N}}$$

b) $$F_N = 525 + 60 - 32.3 = \boxed{553\,\text{N}}$$



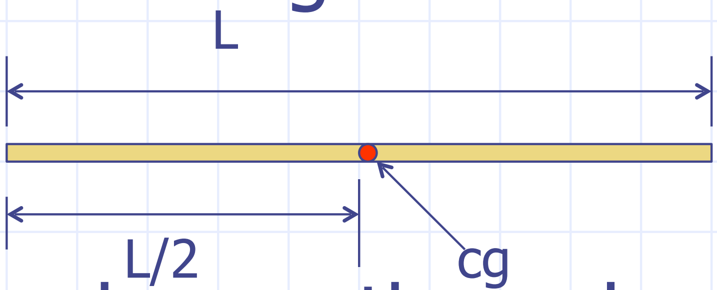
We did not consider the Normal Force when calculating the torques since its lever arm is zero

Center of Gravity



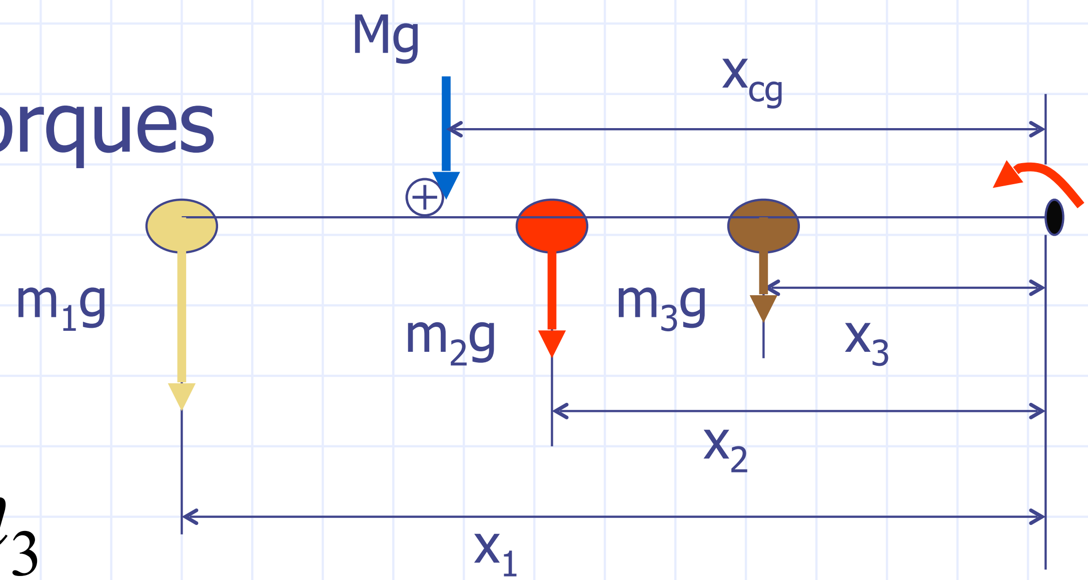
- The point at which the weight of a rigid body can be considered to act when determining the torque due to its weight

- Consider a uniform rod of length L. Its center of gravity (cg) corresponds to its geometric center, L/2.



- Each particle which makes up the rod creates a torque about cg, but the sum of all torques due to each particle is zero

- So, we treat the weight of an extended object as if it acts at one point
- Consider a collection of point-particles on a massless rod
- The sum of the torques



$$m_1 g x_1 + m_2 g x_2 + m_3 g x_3 = M g x_{cg}$$

$$M = m_1 + m_2 + m_3$$

$$\Rightarrow x_{cg} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{M} = x_{cm}$$

Angular Momentum

- In Chapter 2, we defined the linear momentum

$$\vec{p} = m\vec{\mathrm{v}}$$

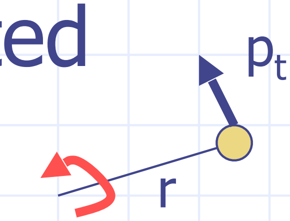
- Analogously, we can define Angular Momentum

$$\vec{L} = I\vec{\omega}$$

- Since ω is a vector, **L** is also a vector

- **L** has units of kg m² /s

- The linear and angular momenta are related



$$L = I\omega = (mr^2)\left(\frac{\mathbf{v}_t}{r}\right) = rm\mathbf{v}_t = rp_t$$

- L gives us another way to express the rotational motion of an object
- For linear motion, if an external force was applied for some short time duration, a change in linear momentum resulted $$\vec{F}_{ext}\Delta t = \vec{p}_f - \vec{p}_i$$
- Similarly, if an external torque is applied to a rigid body for a short time duration, its angular momentum will change $$\tau_{ext}\Delta t = L_f - L_i$$
- If $$\tau_{ext} = 0 \quad \textbf{then} \quad \boxed{L_f = L_i}$$
- This is the Principle of Conservation of Angular Momentum

- How to interpret this? Say the moment of inertia of an object can decrease. Then, its angular speed must increase.

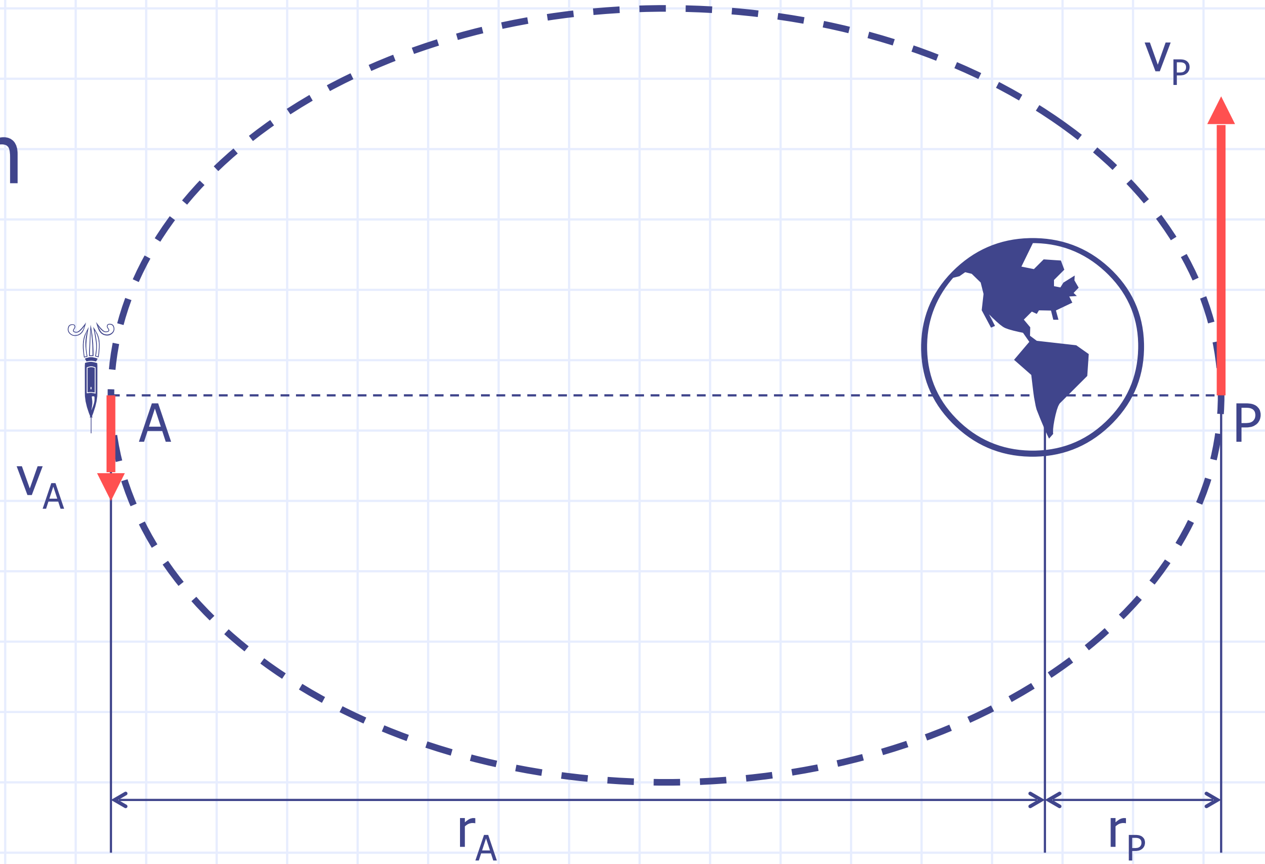
$$I_i > I_f, \qquad L_f = L_i$$

$$I_f \omega_f = I_i \omega_i \Rightarrow \omega_f = \frac{I_i}{I_f} \omega_i > \omega_i$$

Example Problem

For a certain satellite with an apogee distance of $r_A = 1.30 \times 10^7$ m, the ratio of the orbital speed at perigee to the orbital speed at apogee is 1.20. Find the perigee distance r_P. → Not uniform circular motion

- Satellites generally move in elliptical orbits. (Kepler's 1st Law). Also, the tangential velocity is not constant.



- If the satellite is ``circling" the Earth, the furthest point in its orbit from the Earth is called the ``apogee." The closest point the ``perigee." For the Earth circling the sun, the two points are called the ``aphelion" and ``perihelion."

Given: $r_A = 1.30 \times 10^7$ m, $v_P/v_A = 1.20$. Find: r_P ?

Method: Apply Conservation of Angular Momentum. The gravitational force due to the Earth keeps the satellite in orbit, but that force has a line of action through the center of the orbit, which is the rotation axis of the satellite. Therefore, the satellite experiences no external torques.

$$L_A = L_P$$

$$I_A \omega_A = I_P \omega_P$$

$$m r_A^2 \left(\frac{v_A}{r_A} \right) = m r_P^2 \left(\frac{v_P}{r_P} \right)$$

$$r_A v_A = r_P v_P$$

$$r_P = r_A (v_A / v_P)$$

$$= (1.30 \text{x} 10^7)(1/1.20)$$

$$= \boxed{1.08 \text{x} 10^7 \text{ m}}$$

Summary

Translational		Rotational
x	displacement	θ
v	velocity	ω
a	acceleration	α
F	cause of motion	τ
m	inertia	I
$\Sigma F=ma$	2nd Law	$\Sigma \tau=I \alpha$
Fs	work	$\tau \theta$
$1/2mv^2$	KE	$1/2I \omega^2$
$p=mv$	momentum	$L=I \omega$