KEY

PHYS 1211, Fall 2012

Test #1

September 6, 2012

2:00 pm– 3:15 pm

1. ________
2. ________
3. ________
4. ________

Total ________

Name ____________________

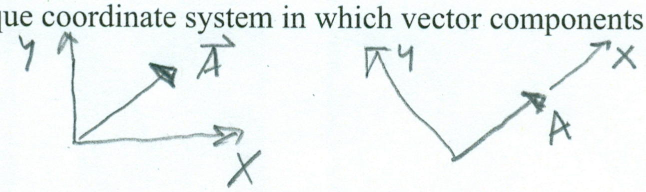
ID ____________________

NOTE: This test consists of one set of conceptual questions and three problems. In working the problems, you <u>must show all of your calculations</u> and your reasoning clearly to receive credit. Be sure to include units in your solutions when required.

1. Conceptual questions (10 points, no calculations required). State whether the following statements are *True* or *False*.

a) There is only one unique coordinate system in which vector components can be added.

False



etc.

b) The addition of a vector and a scalar results in a vector.

False

$\vec{A} + \vec{B} = \vec{C}$ allowed

$A_x + B_x = C_x$ allowed

$\vec{A} + |\vec{B}| =$ not allowed

c) In uniform circular motion, the magnitude of the centripetal (or radial) acceleration a_r is constant.

$a_r = \frac{v^2}{r}$

For UCM, $|\vec{v}|$ = constant,

and $\vec{r}$ = constant,

therefore a_r remains constant

True

2. The height of a helicopter above the ground is given by $h=3.00t^3$, where h is in meters and t is in seconds. After 2.00s, the helicopter releases a small mailbag. How long after its release does the mailbag reach the ground? (30 points)

The helicopters motion is given by

$$y_h(t) = h(t) = 3t^3$$

Its velocity is therefore

$$V_{hy}(t) = \frac{dh(t)}{dt} = 9t^2$$

at $t_0 = 0$, $y_{0h} = 0$

$$V_{0hy} = 9(0)^2 = 0$$

at $t_1 = 2s$,

$$Y_{1h} = 3(2)^3 = 24.0\,m$$

$$V_{1hy} = 9(2)^2 = 36.0\frac{m}{s^2}$$

Now consider mailbag. At $t_1 = 2s$, mailbag is released so, helicopter and mailbag have same height and velocity.

Find t_2, when mailbag hits the ground. Know; $t_1 = 2s$, $y_1 = 24m$, $V_1 = 36\frac{m}{s}$, $y_2 = 0$. Don't know t_2 or V_2. So use kinematic equation

$$Y_2 = y_1 + V_1\Delta t - \frac{1}{2}g\Delta t^2 \quad -(1)$$

where $\Delta t = t_2 - t_1$

(1) becomes

$$\Delta t^2 - \frac{2V_1}{g}\Delta t - \frac{2y_1}{g} = 0$$

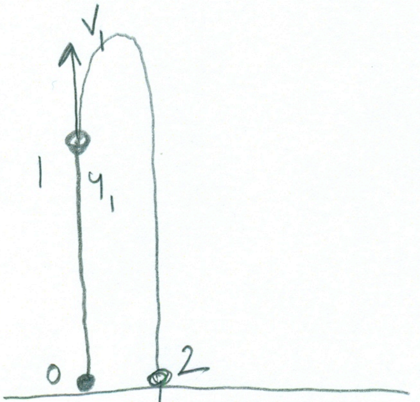
which is a quadratic equation with solution

$$\Delta t = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a} \qquad \text{with } a=1,\ b=-\frac{2V_1}{g},\ c=-\frac{2y_1}{g}$$

$$= \frac{V_1}{g} \pm \sqrt{\left(\frac{V_1}{g}\right)^2 + \frac{2y_1}{g}}$$

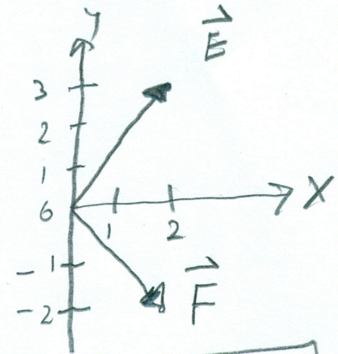
$$= \frac{36}{9.8} \pm \sqrt{\left(\frac{36}{9.8}\right)^2 + \frac{2(24)}{9.8}}$$

$$= 3.673 \pm 4.289 = \boxed{7.962} \text{ or } -0.615$$



$$\boxed{\Delta t = 7.962\,s}$$

3. Let $\vec{E} = 2\hat{i} + 3\hat{j}$ and $\vec{F} = 2\hat{i} - 2\hat{j}$. Find (a) the magnitude of $\vec{E}$, (b) the magnitude and direction of $\vec{A} = \vec{E} + \vec{F}$, and (c) the magnitude and direction of $\vec{B} = -\vec{E} - 2\vec{F}$. (d) Sketch $\vec{A}$ and $\vec{B}$. (30 points)



a) $E = \sqrt{E_x^2 + E_y^2} = \sqrt{2^2 + 3^2}$

$= \sqrt{13} = 3.606 = \boxed{3.6}$

b) $A_x = E_x + F_x = 2 + 2 = 4$

$A_y = E_y + F_y = 3 - 2 = 1$

$A = \sqrt{A_x^2 + A_y^2} = \sqrt{4^2 + 1^2} = \sqrt{17} = 4.123 = \boxed{4.1}$

$\theta_A = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{1}{4}\right) = \boxed{14.0^\circ}$

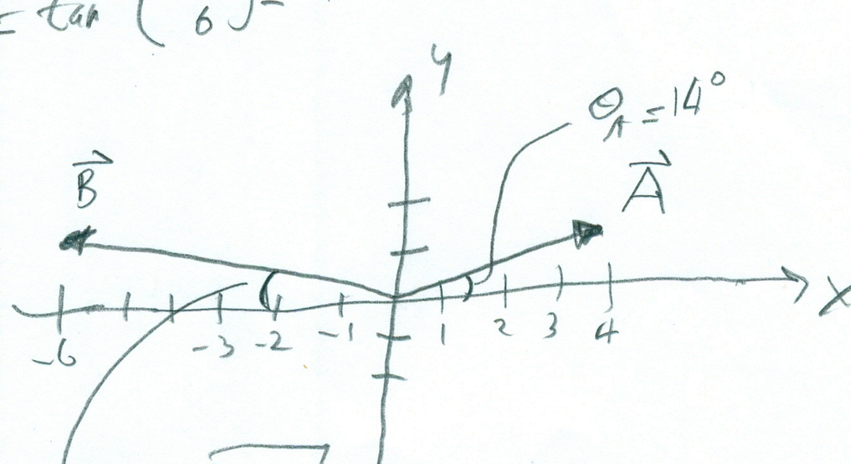
c) $B_x = -E_x - 2F_x = -2 - 2(2) = -6$

$B_y = -E_y - 2F_y = -3 - 2(-2) = 1$

$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(-6)^2 + 1^2} = \sqrt{37} = 6.082$

$\theta_B = \tan^{-1}\left(\frac{B_y}{B_x}\right) = \tan^{-1}\left(\frac{1}{6}\right) = 9.46^\circ$

d)



$\theta_B = \boxed{9.5^\circ}$ w.r.t. negative X-axis.

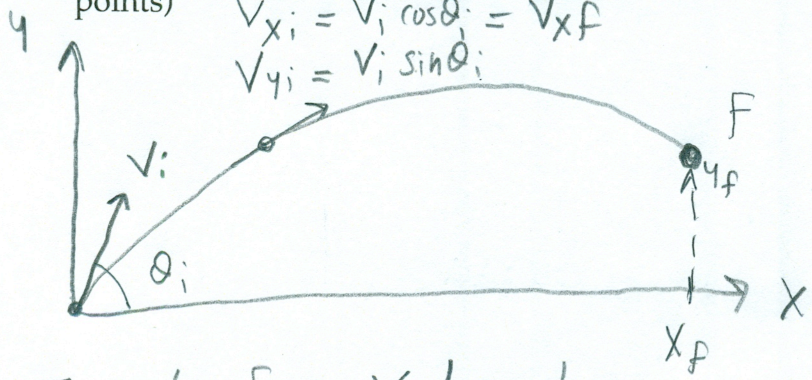
or $\theta_B = 180 - 9.46^\circ = \boxed{170.5^\circ}$ w.r.t X-axis

2D Kinematics for arrow, 1D free-fall for apple

4. An archer shoots an arrow with a velocity of 45.0 m/s at an angle of 50.0° above the horizontal. An assistant standing on level ground 150 m downrange from the launch point throws an apple straight up with the minimum initial speed necessary to meet the path of the arrow. (a) Determine the time for the arrow to hit the apple. (b) Determine the height, above the launch point of the arrow, when the arrow and apple collide. (c) Assuming that the arrow and apple are launched from the same height, what is the initial speed of the apple? (30 points)

$V_{xi} = V_i \cos\theta_i = V_{xf}$

$V_{yi} = V_i \sin\theta_i$



arrow: $t_i = 0$, $V_i = 45\,m/s$, $\theta_i = 50°$, $X_i = 0$, $y_i = 0$

$X_f = 150\,m$, $y_f = ?$

$t_f = ?$, $V_{yf} = ?$

a) Find t_f from X-direction

$$X_f = X_i + \frac{1}{2}(V_{xi} + V_{xf})\,t_f = V_{xi}\,t_f$$

or $$t_f = \frac{X_f}{V_{xi}} = \frac{X_f}{V_i \cos\theta_i} = \frac{150\,m}{\left(45\frac{m}{s}\right)\cos 50°} = 5.1857 = \boxed{5.19\,s}$$

b) Find y_f (Don't know V_{yf}), use

$$y_f = y_i + V_{yi}\,t_f - \frac{1}{2} g\,t_f^2 = 0 + V_i \sin\theta_i\,t_f - \frac{1}{2} g\,t_f^2$$

$$= \left(45\frac{m}{s}\right)\sin 50°(5.1857) - \frac{1}{2}(9.8)(5.1857)^2 = 46.993$$

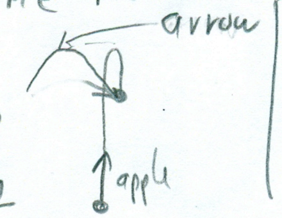
$$= \boxed{47.0\,m}$$

c) Now, one could launch the apple with the same y-component of the arrow's initial velocity and they would both have the same motion in the y-direction. This would be

$V_{iy} = V_i \sin\theta_i$

$= 45 \sin 50°$

$= 34.5\frac{m}{s}$



However, this is not the minimum initial speed. Instead, we want the final apple velocity to be $V_f = 0$, or

$$V_{yf}^2 = V_{yi}^2 - 2g(y_f - y_i) \quad \text{or}$$

$$V_{yi} = \sqrt{2g\,y_f} = \sqrt{2(9.8)(47)} = \boxed{30.4\frac{m}{s}}$$