PHYS 1211, Fall 2012

Test #4, version 1

November 27, 2012

2:00 pm– 3:15 pm

Name ______________ KEY

ID ______________

1. ________
2. ________
3. ________
4. ________

Total ________

NOTE: This test consists of one set of conceptual question and three problems. In working the problems, you <u>must show all of the algebra and calculations</u> and your reasoning clearly to receive credit. Be sure to include units in your solutions when required.

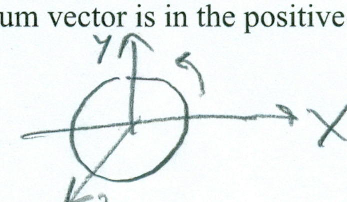
1. Conceptual questions (10 points, no calculations required). State whether the following statements are *True* or *False*.

A. For a solid uniform sphere and a thin hoop, each of mass M and radius R and rotating about their respective centers of mass, the moment of inertia of the hoop is larger than that of the sphere.

True more mass at R and $I = \Sigma m_i r_i^2$

B. For a disk rotating counter-clockwise in the x-y plane through an axis about its center of mass, the direction of the angular momentum vector is in the positive z direction

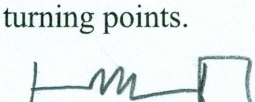
True $\vec{L} = L\hat{k}$

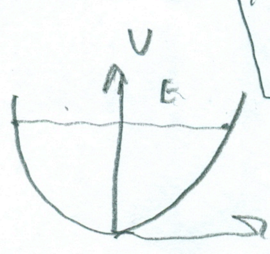


C. For a mass-spring system oscillating about its equilibrium point at x=0, the magnitude of the force acting on the mass is greatest at the turning points.

True

$F = ma = m[-A\omega^2 \cos(\omega t + \phi_0)]$

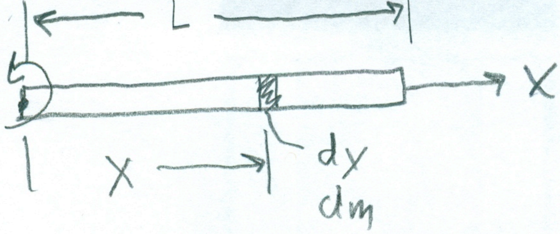




or $F_x = -\frac{dU}{dx}$, slope greatest at turning points

2. a) Use the integral relation to **derive** the moment of inertia of a uniform rod of length L and mass M rotating about one end of the rod. b) If the rod has a mass of 2.0 kg and a length of 2.0 meters, what is its angular momentum vector if it rotates counter-clockwise about one end with an angular speed of 15 rpm? (30 points)

a)



linear density

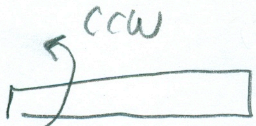
$$\lambda = \frac{M}{L} \Rightarrow M = \lambda L$$

or $dm = \lambda dx$

$$I_{end} = \int_0^L x^2 dm = \lambda \int_0^L x^2 dx = \lambda \left[\frac{x^3}{3}\right]_0^L$$

$$= \lambda\left[\frac{L^3}{3} - \frac{0^3}{3}\right] = \frac{M}{L}\left[\frac{L^3}{3}\right] = \boxed{\frac{1}{3}ML^2}$$

b)



$$\omega = 15 \text{ rpm} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} \times \frac{1 \text{ min}}{60 \text{ sec}}$$

$$= 1.57 \frac{\text{rad}}{s}$$

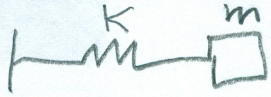
$$\vec{L} = I\vec{\omega}$$

$$= \left(\frac{1}{3}ML^2\right)(\omega \hat{k}) = \frac{1}{3}(2 \text{ kg})(2 \text{ m})^2\left(1.57 \frac{\text{rad}}{s}\right)\hat{k}$$

$$\boxed{\vec{L} = 4.19 \text{ kg}\frac{m^2}{s}\hat{k}}$$

or ccw

3. The position of a 50 g oscillating mass attached to a spring is given by $x(t) = (2.0\text{ cm})\cos(10t - \pi/4)$ where t is in s. Determine: a) the period of oscillation, b) the phase constant, c) the initial conditions (x and v at t=0), and d) the maximum speed. (30 points)



a) $\omega = \sqrt{\frac{k}{m}}$

$x(t) = A\cos(\omega t + \phi_0)$

$\omega = 10\ \frac{\text{rad}}{\text{s}} = 2\pi f = \frac{2\pi}{T}$

$T = \frac{2\pi}{\omega} = \frac{2\pi\text{ rads}}{10\text{ rads/s}}$

$\boxed{T = 0.628\text{ s}}$

b) $\boxed{\phi_0 = -\frac{\pi}{4}\text{ rads}}$

c) $X(0) = A\cos\phi_0 = 2.0\text{ cm}\cos\left(-\frac{\pi}{4}\right)$

$\boxed{X(0) = 1.41\times10^{-2}\text{ m}}$

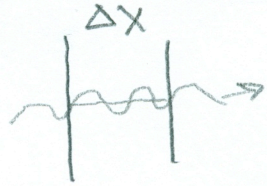
$V(t) = -A\omega\sin(\omega t + \phi_0)$

$V(0) = -A\omega\sin\phi_0 = -(2\times10^{-2}\text{ m})\left(10\ \frac{\text{rad}}{\text{s}}\right)\sin\left(\frac{-\pi}{4}\right)$

$\boxed{V(0) = 1.41\times10^{-1}\ \frac{\text{m}}{\text{s}}}$

d) $V_{max} = A\omega = (2.0\text{ cm})\left(10\ \frac{\text{rad}}{\text{s}}\right) = 20\ \frac{\text{cm}}{\text{s}} = \boxed{0.2\ \frac{\text{m}}{\text{s}}}$

4. a) How long does it take light to travel through a 3.0-mm thick piece of window glass? b) Through what thickness of water could light travel in the same amount of time? c) What is the wavelength of light in parts a) and b)? d) If a sound wave of frequency 500 Hz traveled through the window glass and water, what would be its wavelengths in each material? Take the index of refraction for water and glass to be 1.33 and 1.5, respectively. The speed of sound in water and glass are 1480 m/s and 3960 m/s, respectively. Take the speed of light in a vacuum to be 3.0×10^8 m/s. (30 points)



a) light $n = \frac{c}{v} \Rightarrow v = \frac{c}{n}$

$\Delta x = vt = \frac{c}{n} t$

or $t_{glass} = \frac{\Delta x n}{c} = \frac{(3 \times 10^{-3} m)(1.5)}{3 \times 10^8 m/s} = \boxed{1.5 \times 10^{-11} s}$ glass

b) $t_W = \Delta X_W \frac{n_W}{c} = t_{glass}$

or $\Delta X_W = \frac{t c}{n_W} = \frac{(1.5 \times 10^{-11} s)(3 \times 10^8 \frac{m}{s})}{1.33}$

$= \boxed{3.4 \times 10^{-3} m} = 3.4$ mm

c) $v = \lambda f = \frac{c}{n}$

or $\lambda = \frac{c}{fn}$

$\lambda_{water} = \frac{3 \times 10^8 m/s}{f \, 1.33} = \boxed{\frac{2.26 \times 10^8}{f(Hz)} m}$

$\lambda_{glass} = \frac{3 \times 10^8 m/s}{f \, 1.5} = \boxed{\frac{2.0 \times 10^8}{f(Hz)} m}$

d) $V_{sound} = \lambda f$

$\lambda = \frac{V_{sound}}{f}$

$\lambda_{glass} = \frac{3960 \frac{m}{s}}{500 Hz} = \boxed{7.92 m}$

$\lambda_{water} = \frac{1480 m/s}{500 Hz} = \boxed{2.96 m}$

Bonus, since I forgot to give a Frequency