KEY

**PHYS 1211 Fall 2019 Test 1**
September 17, 2019

**Name** _____ **Student ID** _____ **Score** _____

**Note: This test consists of one set of conceptual questions, five problems, and a bonus problem. For the problems, you *must show all* of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last page.**

**Problem 1. Conceptual questions.** State whether the following statements are *True* or *False*. (10 points total, no calculations required)

(a) In uniform circular motion, the magnitude of the tangential velocity is constant.

True

(b) For projectile motion, the velocity in the horizontal direction changes as a consequence of the acceleration due to gravity.

False , $V_{xi} = V_{xf}$ since $a_x = 0$

(c) For the two vectors $\vec{A}$ and $\vec{B}$, the operation $\vec{A}/\vec{B}$ is valid.

False Cannot divide vectors

$x_f$, $x_i = 0$

**Problem 2.** A Porsche challenges a Honda to a 400 m race. Because the Porsche's acceleration is 3.5 m/s$^2$ larger than the Honda's 3.0 m/s$^2$ acceleration, the Honda gets a 1.0 s head start. Who wins? By how many seconds? (15 points total)

$a_H = 3.0 \frac{m}{s^2}$ Honda: $x_f = x_i^{\,0} + v_{xi}^{\,0} t_f + \frac{a_H}{2} t_f^2$

$a_P = 6.5 \frac{m}{s^2}$ $x_f = \frac{a_H}{2} t_f^2$

$v_{iH} = 0 = v_{iP}$ $t_f = \sqrt{\frac{2x_f}{a_H}} = \sqrt{\frac{2(400m)}{3\,m/s^2}} = \boxed{16.33\,s}$

Porsche:

$x_f = \frac{a_P}{2}(t_f - t_i)^2$ $t_f = \sqrt{\frac{2x_f}{a_P}} + t_i = \sqrt{\frac{2(400)}{6.5}} + 1$

$t_f - t_i = \sqrt{\frac{2x_f}{a_P}} \Rightarrow$ $= \boxed{12.09\,s}$ $\boxed{\text{Porsche wins by } 4.24\,s}$

**Problem 3.** The position of a particle as a function of time is given by $\vec{r} = (6.0\hat{i} + 5.0\hat{j})t^2$ m, where $t$ is in seconds. (a) Find an expression for the particle's velocity $\vec{v}$ as a function of time. (b) What are the units for the constant 6.0? (15 points total)

a) $\frac{d\vec{r}}{dt} = \frac{d}{dt}(6.0t^2\hat{i} + 5.0t^2\hat{j})\,m$

$\boxed{\vec{v} = (12.0t\,\hat{i} + 10.0t\,\hat{j})\,m/s}$

b) $\left(6.0 \frac{m}{s^2}\right)(t^2 s^2) = 6.0t^2\,m$

$\Rightarrow \boxed{\frac{m}{s^2}}$

**Problem 4.** The angular velocity of a motor is $\omega = (20 - \frac{1}{2}t^2)$ rad/s, where $t$ is in seconds. Through what angle does the motor turn from $t_i = 0$ to $t_f = 11$ s? (15 points total)

$\omega = \frac{d\theta}{dt}$

$d\theta = \omega dt$

$\int_{\theta_i}^{\theta_f} d\theta = \int_{t_i}^{t_f} \omega dt$

$\theta\Big|_{\theta_i}^{\theta_f} = \int_{t_i}^{t_f} (20 - \frac{t^2}{2}) dt$

$\theta_f - \theta_i = \left[20t - \frac{t^3}{6}\right]\Big|_{t_i=0}^{t_f=11}$

$\theta_f = 20(11) - \frac{(11)^3}{6} = \boxed{-1.83 \text{ rad}}$

**Problem 5.** Your roommate is working on his bicycle and has the bike upside down. He spins the 60.0-cm-diameter wheel, and you notice that a pebble stuck in the tread goes by three times every second. What are the pebble's angular speed, angular acceleration, tangential speed, and radial acceleration? (15 points total)

$r = 0.3 \text{ m}$

$T = \frac{1 \text{ s}}{3}$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{1/3} = 6\pi \frac{\text{rad}}{\text{s}} = \boxed{18.85 \frac{\text{rad}}{\text{s}}}$

$\boxed{\alpha = 0}$, since $\omega$ = constant

$v_t = r\omega = (0.3 \text{ m})(18.85 \text{ rad/s}) = \boxed{5.65 \text{ m/s}}$

$a_r = \frac{v_t^2}{r} = \frac{(5.6547)^2}{0.3} = \boxed{106.6 \frac{\text{m}}{\text{s}^2}}$

$$|\vec{V}_f| = \sqrt{V_{fx}^2 + V_{fy}^2} = \sqrt{8.86^2 + 27.65^2} = \boxed{29.0\ m/s}$$

$$\theta = \tan^{-1}\left(\frac{V_{fy}}{V_{fx}}\right) = \tan^{-1}\left(\frac{-8.86}{27.65}\right) = \boxed{-17.8^\circ}$$

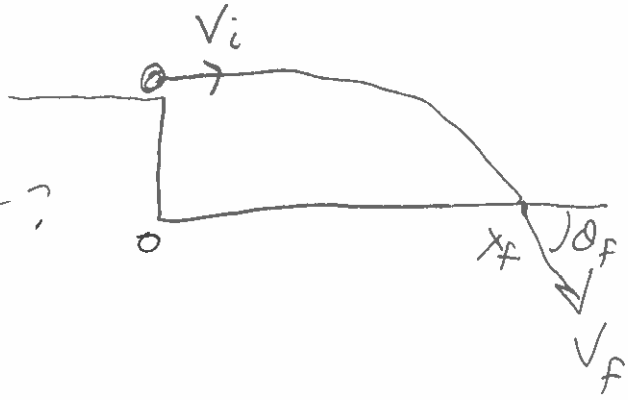
**Problem 6.** A friend of yours on the UGA baseball team wants to determine her pitching speed. You have her stand on a ledge of height 4.00 m above the ground and throw the ball horizontally. The ball lands a horizontal distance of 25.0 m away. What was her pitching speed? What is the total time the ball is in the air? Give the final velocity of the baseball in component form. Find the magnitude and direction angle of the final velocity. (30 points total)

$x_i = 0$, $y_i = 4.0\ m$

$x_f = 25.0\ m$, $y_f = 0\ m$

$V_{iy} = 0$, $V_{ix} = ?$, $t_f = ?$

$V_{fy} = ?$




First find the time

$$y_f = y_i + V_{iy} t_f - \frac{1}{2} g t_f^2$$

$$0 = y_i + 0 - \frac{1}{2} g t_f^2$$

$$t_f = \sqrt{\frac{2y_i}{g}} = \sqrt{\frac{2(4.0\ m - 0)}{9.8\ m/s^2}} = \boxed{0.904\ s}$$

$$x_f = x_i + \frac{1}{2}(V_{xi} + V_{xf})\, t_f \qquad \text{but } V_{xi} = V_{xf}$$

$$x_f = 0 + V_{xi} t_f$$

$$V_{xi} = \frac{x_f}{t_f} = \frac{25.0\ m}{0.904\ s} = \boxed{27.65\ m/s}$$

$$V_{yf} = V_{yi} - g t_f = 0 - 9.8\frac{m}{s^2}(0.904\ s) = \boxed{-8.86\ \frac{m}{s}}$$

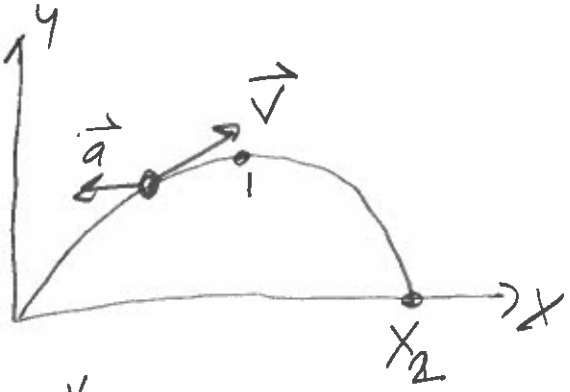
$$\vec{V}_f = \langle -8.86, 27.65 \rangle\ m/s$$

should be <27.65,-8.86>

**Bonus Problem.** A projectile is launched from ground level at an angle $\theta$ and speed $v_0$ into a headwind that causes a constant horizontal acceleration of magnitude $a$ opposite the direction of motion. (a) Derive an expression in terms of $a$, $g$, and $v_0$ for the launch angle that gives maximum range. (b) What is the angle for maximum range if $a$ is 10% of $g$? (5 points total)

$V_{0x} = V_0 \cos\theta$ $\quad x_0 = 0,\ y_0 = 0 = y_2$

$V_{0y} = V_0 \sin\theta$




$X_2 = X_0 + V_{0x} t_2 - \frac{1}{2} a t_2^2$

Motion in y-direction is unaffected

$V_{1y} = 0$, $V_{1y} = V_{0y} - g t_1 \Rightarrow t_1 = \frac{V_{0y}}{g} = \frac{V_0 \sin\theta}{g}$

total time is $t_2 = \frac{2V_0 \sin\theta}{g}$

Therefore

$$X_2 = V_0 \cos\theta \left(\frac{2V_0 \sin\theta}{g}\right) - \frac{1}{2} a \left(\frac{2V_0 \sin\theta}{g}\right)^2$$

$$= \frac{2V_0^2 \cos\theta \sin\theta}{g} - \frac{2V_0^2 \sin^2\theta\, a}{g^2} = \frac{V_0^2}{g}\left[\sin 2\theta - \frac{2a}{g}\sin^2\theta\right]$$

to find max.

$$\frac{dX_2}{d\theta} = \frac{V_0^2}{g}\left[2\cos 2\theta - \frac{2a}{g}(2\sin\theta\cos\theta)\right]$$

$$= \frac{2V_0^2}{g}\left[\cos 2\theta - \frac{a}{g}\sin 2\theta\right] = 0$$

or $\cos 2\theta = \frac{a}{g}\sin 2\theta$

$\cot 2\theta = \frac{a}{g}$

$\boxed{\theta = \frac{1}{2}\tan^{-1}\left(\frac{g}{a}\right)}$

$a = \frac{g}{10}$

$\theta = \frac{1}{2}\tan^{-1}\left(\frac{g}{g/10}\right)$

$= \frac{1}{2}\tan^{-1}(10)$

$= \boxed{42.14^\circ}$