KEY

PHYS 1211 Fall 2019 Test 2
October 10, 2019

Name ______________________________ **Student ID** ______________________________ **Score** ________

Note: This test consists of one set of conceptual questions, five problems, and a bonus problem. For the problems, you *must show all* of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last pages.

Problem 1. Conceptual questions. State whether the following statements are *True* or *False*. (10 points total, no calculations required)

(a) Newton's 1st Law implies that the laws of physics are the same for a reference frame moving at a constant velocity and a reference frame at rest.

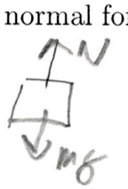
True $\Sigma \vec{F} = 0$

(b) For a UPS package at rest on an inclined ramp, the friction force it experiences is given in general by $\mu_s N$.

False $\mu_s N = f_s^{max}$ ← only

(c) For a coffee mug at rest on a table, the normal force and gravitational force acting on the mug are related by Newton's 3rd Law.

False



related by Newton's 2nd law

(d) The planet Mercury's orbital speed is larger than Neptune's orbital speed.

True $V = \sqrt{\frac{G M_{sun}}{r}}$ $r_M << r_N$

$V_M >> V_N$

Problem 2. Given the vectors $\vec{A} = 5\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\vec{B} = 3\hat{\imath} - 4\hat{\jmath} + 1\hat{k}$, determine (a) $\vec{A} + \vec{B}$ and $\vec{A} \cdot \vec{B}$. (15 points total)

a) $\vec{A} + \vec{B} = (5+3)\hat{\imath} + (2-4)\hat{\jmath} + (-3+1)\hat{k}$

$\vec{A} + \vec{B} = \boxed{8\hat{\imath} - 2\hat{\jmath} - 2\hat{k}} = \langle 8, -2, -2 \rangle$

b) $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

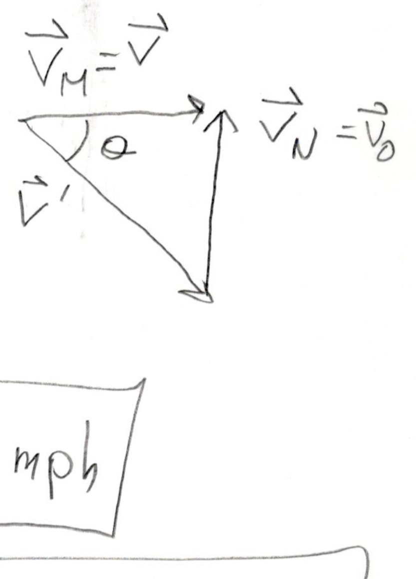
$= (5)(3) + (2)(-4) + (-3)(1)$

$= 15 - 8 - 3 = \boxed{4}$

Problem 3. Neil, driving north at 60.0 mph, and Madhurya driving east at 50.0 mph, are approaching an intersection. What is Madhurya's velocity relative to Neil's reference frame? (i.e., as seen by Evan who is a passenger in Neil's car). (15 points total)

$\vec{V}_N = 60 \text{ mph}\,\hat{\jmath}$, $\vec{V}_M = 50 \text{ mph}\,\hat{\imath}$

Using the Galilean Transformation



$\vec{V}' = \vec{V} - \vec{V}_0$

$\vec{V} = \vec{V}_M$, $\vec{V}_0 = \vec{V}_N$

Find $\vec{V}' = \vec{V} - \vec{V}_0$

$|\vec{V}'| = \sqrt{V_M^2 + V_0^2} = \sqrt{50^2 + 60^2} = \boxed{78.1 \text{ mph}}$

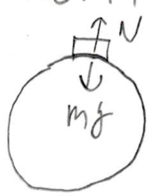
$\theta = \tan^{-1}\left(\frac{V_N}{V_M}\right) = \tan^{-1}\left(\frac{60}{50}\right) = \boxed{50.2^\circ \text{ South of East}}$

or $= -50.2^\circ$ or 309.8°

$$T = 1\,\text{day} \times \frac{24\,\text{hrs}}{1\,\text{day}} \times \frac{60\,\text{mins}}{1\,\text{hr}} \times \frac{60\,\text{secs}}{1\,\text{min}} = 8.64\times10^{4}\,\text{s}$$

Problem 4. Adam, who has a mass of 75.0 kg, weighs himself with a scale at the north pole and at the equator. (a) Which scale reading is higher? (b) By how much? (Assume the Earth is spherical with a radius of 6.37×10^3 km. Flat-earthers still have to do this problem). (15 points total)

a) North pole



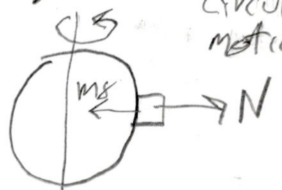
$$\Sigma F_y = ma_y = 0$$

$$N - mg = 0$$

$$N = mg = (75.0\,\text{kg})(9.8\,\tfrac{m}{s^2})$$

$$= 735\,N$$

Equator (Uniform circular motion)



$$\Sigma F_r = ma_r$$

$$mg - N = \frac{mv^2}{r}$$

$$N = mg - \frac{mv^2}{r}$$

$N < mg$ $\boxed{\text{North pole higher}}$

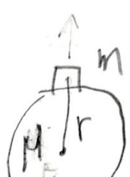
b) $$V = \frac{2\pi r}{T}$$

$$\frac{mv^2}{r} = \frac{m4\pi^2 r}{T^2}$$

$$= \frac{(75)4\pi^2(6.37\times10^6)}{(8.64\times10^4)^2}$$

$$= 2.53\,N$$

Problem 5. Haley lands Elon Musk's Starship on a newly discovered planet. The planet has a radius twice as large as Earth's and a mass five times as large as Earth's. What is the free-fall acceleration on the planet's surface? (15 points total)



$$mg = \frac{GMm}{r^2}$$

or

$$g = \frac{GM}{r^2}$$

on Earth

$$mg_E = \frac{GM_E m}{r_E^2}$$

$$g_E = \frac{GM_E}{r_E^2}$$

$$r_p = 2r_E$$

$$M_p = 5M_E$$

on Planet

$$g_p = \frac{GM_p}{r_p^2}$$

$$= \frac{G(5M_E)}{(2r_E)^2}$$

$$= \frac{5}{4}\left(\frac{GM_E}{r_E^2}\right)$$

$$= \boxed{\frac{5}{4}g_E = 12.3\,\frac{m}{s^2}}$$

Problem 6. (a) Starting with Newton's 2nd Law, derive the work-kinetic energy theorem. (b) Now imagine that Justin throws a 20.0 g particle to the left at 30.0 m/s. A force acts on the particle and causes it to move to the right at 30.0 m/s. How much work is done by the force on the particle? (c) Consider the same situation in part (b), but imagine that the force acts such as to bring the particle to rest over a distance of 0.001 m. Assuming that the force is constant, determine its magnitude and direction. (30 points total)

a) $\vec{F}_{net} = \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$, use 1D

chain rule

$F_x = m\frac{dv_x}{dt} = m\frac{dv_x}{dx}\frac{dx}{dt} = m\frac{dv_x}{dt}v_x$

or $F_x\,dt = m\,v_x\,dv_x$

or $\int_{t_i}^{t_f} F_x\,dt = m\int_{v_{xi}}^{v_{xf}} v_x\,dv_x$ for constant m

or $W = m\left[\frac{v_x^2}{2}\right]_{v_{xi}}^{v_{xf}} = \frac{1}{2}m\left[v_{xf}^2 - v_{xi}^2\right]$

$W = \frac{1}{2}m v_{xf}^2 - \frac{1}{2}m v_{xi}^2 = \boxed{K_f - K_i = W}$

b) $W = \frac{1}{2}m\left[v_{x_f}^2 - v_{x_i}^2\right] = \frac{1}{2}m\left[\left(30\frac{m}{s}\right)^2 - \left(30\frac{m}{s}\right)^2\right] = \boxed{0}$

c) $W = F_x\Delta x = \frac{1}{2}m\cancel{v_{x_f}^2} - \frac{1}{2}m v_{x_i}^2 = -\frac{1}{2}m v_{x_i}^2$ (with $v_{x_f} = 0$)

$F_x = -\frac{m v_{x_i}^2}{2\Delta x} = -\frac{(0.02\,kg)(30\,m/s)^2}{2(-0.001\,m)} = 9000\,N$

or $\boxed{\vec{F}_x = +9000\,\hat{i}\,N}$

Bonus Problem. A block of mass m is at rest at the origin at $t = 0$. It is pushed with constant force F_0 from $x = 0$ to $x = L$ across a horizontal surface whose coefficient of kinetic friction is $\mu_k = \mu_0(1 - x/L)$. That is, the coefficient of kinetic friction decreases from μ_0 at $x = 0$ to zero at $x = L$. (a) Obtain the general expression for the acceleration

$$a_x = v_x \frac{dv_x}{dx}. \tag{1}$$

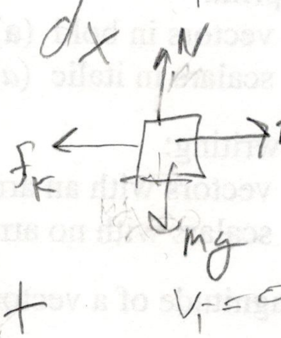
(b) Find an expression for the block's speed as it reaches $x = L$. (5 points total)

a) Similar to problem 6, use the chain rule

$$a_x = \frac{dv_x}{dt} = \frac{dv_x}{dx}\frac{dx}{dt} = v_x \frac{dv_x}{dx}$$

$\Sigma F_y = N - mg = 0$

$N = mg$



b) $\Sigma F_x = F_0 - f_k = ma_x$

$= F_0 - \mu_k N$

$= F_0 - \mu_0\left(1 - \frac{x}{L}\right)mg = F_{net}$

$v_i = 0$

Use work-energy theorem

$$W = K_f - K_i = \frac{1}{2}mv_f^2$$

$$W = \int_{x_i=0}^{x_f=L} \left(F_0 - \mu_0 mg\left(1 - \frac{x}{L}\right)\right)dx$$

$$= F_0 L - \mu_0 mgL + \int_0^L \frac{\mu_0 mg}{L}x\,dx = [F_0 - \mu_0 mg]L + \frac{\mu_0 mg}{L}\left[\frac{x^2}{2}\right]_0^L$$

$$= [F_0 - \mu_0 mg]L + \frac{\mu_0 mg}{\cancel{L}}\frac{L^{\cancel{2}}}{2} = \left(F_0 - \frac{\mu_0 mg}{2}\right)L = \frac{1}{2}mv_f^2$$

or $$\boxed{v_f = \sqrt{\frac{2F_0 L}{m} - \mu_0 gL}}$$