Chapter 11: Collisions (revisited)

- Return to the boy and the raft conservation of momentum problem. But let's assume that the boy misses the raft.
- Then, the final velocities of the boy and raft are not equal: $v_{f1} \neq v_{f2}$. We then have two unknowns with the conservation of momentum equation (in one-dimension) given by:

$$m_1 \mathrm{v}_{i1} = m_1 \mathrm{v}_{f1} + m_2 \mathrm{v}_{f2}$$

• We need another equation!

- We can use the Energy Principle (COE), applied to the system (neglect rest energy). No change in y – so only K.

$$E_i = E_f$$

$$\tfrac{1}{2} m_1 \mathbf{v}_{i1}^{\mathbf{2}} = \tfrac{1}{2} m_1 \mathbf{v}_{f1}^{\mathbf{2}} + \tfrac{1}{2} m_2 \mathbf{v}_{f2}^{\mathbf{2}}$$

- From original conservation of momentum equation, solve for v_{f2}. Then substitute into conservation of energy equation.

$$\mathbf{v}_{f2} = \frac{m_1}{m_2}(\mathbf{v}_{i1} - \mathbf{v}_{f1})$$

$$\tfrac{1}{2} m_1 \mathbf{v}_{i1}^{\mathbf{2}} = \tfrac{1}{2} m_1 \mathbf{v}_{f1}^{\mathbf{2}} + \tfrac{1}{2} m_2 \left(\frac{m_1^2}{m_2^2} (\mathbf{v}_{i1} - \mathbf{v}_{f1})^2 \right)$$

$$\frac{1}{2}m_1\left(\mathbf{v}_{i1}^2-\mathbf{v}_{f1}^2\right)=\frac{1}{2}\frac{m_1^2}{m_2}\left(\mathbf{v}_{i1}-\mathbf{v}_{f1}\right)^2$$

Here is a trick!

$$\frac{1}{2}m_1\left(\mathbf{v}_{i1}-\mathbf{v}_{f1}\right)\left(\mathbf{v}_{i1}+\mathbf{v}_{f1}\right)$$

$$=\frac{1}{2}\frac{m_1^2}{m_2}\left(\mathbf{v}_{i1}-\mathbf{v}_{f1}\right)\left(\mathbf{v}_{i1}-\mathbf{v}_{f1}\right)$$

$$\mathbf{v}_{i1}+\mathbf{v}_{f1}=\frac{m_1}{m_2}\left(\mathbf{v}_{i1}-\mathbf{v}_{f1}\right)$$

$$m_2\mathbf{v}_{i1}+m_2\mathbf{v}_{f1}=m_1\mathbf{v}_{i1}-m_1\mathbf{v}_{f1}$$

$$\mathbf{v}_{f1}\left(m_1+m_2\right)=\mathbf{v}_{i1}\left(m_1-m_2\right)$$

$$\boxed{\mathbf{v}_{f1}=\left(\frac{m_1-m_2}{m_1+m_2}\right)\mathbf{v}_{i1}}$$

$$\mathbf{v}_{f2} = \frac{m_1}{m_2}(\mathbf{v}_{i1} - \mathbf{v}_{f1})$$

Return to momentum equation

$$\mathbf{v}_{f2} = \frac{m_1}{m_2}\left(\mathbf{v}_{i1} - \frac{m_1 - m_2}{m_1 + m_2}\mathbf{v}_{i1}\right)$$

$$= \left(\frac{m_1(m_1 + m_2) - m_1(m_1 - m_2)}{m_2(m_1 + m_2)}\right)\mathbf{v}_{i1}$$

$$= \left(\frac{m_1 m_1 + m_1 m_2 - m_1 m_1 + m_1 m_2}{m_2(m_1 + m_2)}\right)\mathbf{v}_{i1}$$

$$= \left(\frac{2 m_1 m_2}{m_2(m_1 + m_2)}\right)\mathbf{v}_{i1} = \boxed{\frac{2 m_1}{m_1 + m_2}\mathbf{v}_{i1} = \mathbf{v}_{f2}}$$

Use numerical data from boy and raft example:

$$\mathbf{v}_{f1} = \left(\frac{45-12}{45+12}\right)5.1 = 2.95\,\frac{\mathbf{m}}{\mathbf{s}}$$

$$\mathbf{v}_{f2} = \left(\frac{2*45}{45+12}\right)5.1 = 8.1\,\frac{\mathbf{m}}{\mathbf{s}}$$

$$p_{f1} = m_1\mathbf{v}_{f1} = (45)(2.95) = 133\,\mathbf{kg}\,\frac{\mathbf{m}}{\mathbf{s}}$$

$$p_{f2} = m_2\mathbf{v}_{f2} = (12)(8.1) = 97\,\mathbf{kg}\,\frac{\mathbf{m}}{\mathbf{s}}$$

$$p_{i1} = m_1\mathbf{v}_{i1} = (45)(5.1) = 230\,\mathbf{kg}\,\frac{\mathbf{m}}{\mathbf{s}}$$

Momentum is conserved!

- In almost all of the 2-body problems we will consider, the total momentum will be conserved

- The total mechanical energy may or may not be conserved

- Two kinds of collisions:

1. Elastic – kinetic energies conserved (special case)
   example - boy misses raft
2. Inelastic – kinetic energies not conserved (general) example - boy lands on raft (completely inelastic)

(total energy always conserved in fundamental particle collisions)