Chapter 10 (Cont'd)

- Work done by the spring

$$W_s = F_{s,avg}\cos\phi\, s = F_{s,avg}\cos 0^\circ\ s$$

$$= -\tfrac{1}{2}k(x_f + x_i)(x_f - x_i) = -\tfrac{1}{2}k(x_f^2 - x_i^2)$$

$$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 \quad = -\Delta U$$

$$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 = U_{s,i} - U_{s,f}$$

$$U_{elastic} = U_s = \tfrac{1}{2}kx^2$$

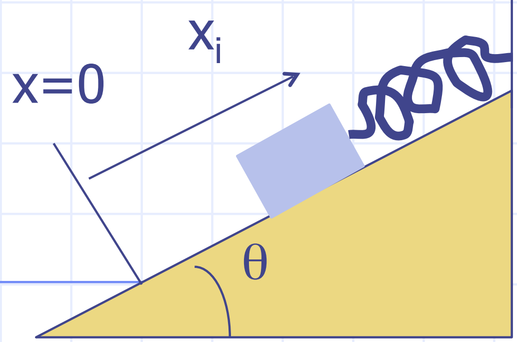
Units of N/m m² = N m = J

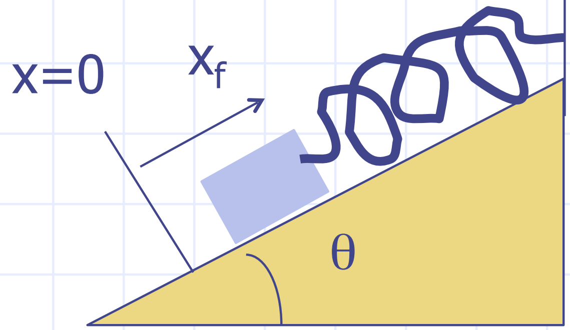
- Total potential energy is

$$U_{total} = U_g + U_s = mgy + \tfrac{1}{2}kx^2$$

Example Problem

A block (m = 1.7 kg) and a spring (k = 310 N/m) are on a frictionless incline (θ = 30°). The spring is compressed by x_i = 0.31 m relative to its unstretched position at x = 0 and then released. What is the speed of the block when the spring is still compressed by x_f = 0.14 m?





Given: m=1.7 kg, k=310 N/m, θ=30°, x_i=0.31 m, x_f=0.14 m, frictionless

Method: no friction, so we can use conservation of energy

$$E = \tfrac{1}{2} m \mathrm{v}^2 + mgh + \tfrac{1}{2} k x^2$$

Initially

$$\mathrm{v}_\mathrm{i} = 0, h_i = x_i \sin\theta$$

$$E_i = mgx_i \sin\theta + \tfrac{1}{2} k x_i^2$$

Finally $h_f = x_f \sin\theta$, find v_f

$$E_f = \tfrac{1}{2}m\mathrm{v}_f^2 + mgx_f\sin\theta + \tfrac{1}{2}kx_f^2$$

$$E_f = E_i$$

$$\tfrac{1}{2}m\mathrm{v}_f^2 + mgx_f\sin\theta + \tfrac{1}{2}kx_f^2 = mgx_i\sin\theta + \tfrac{1}{2}kx_i^2$$

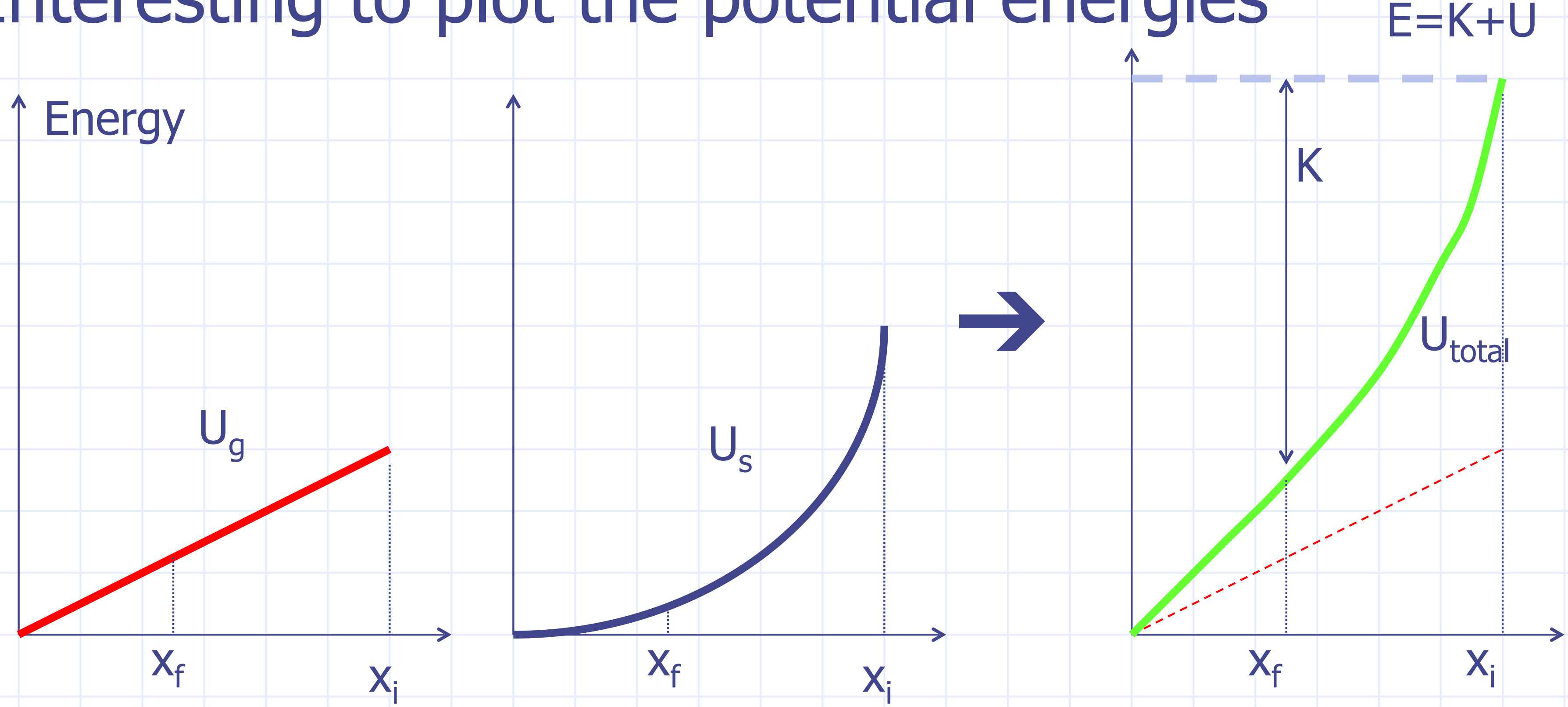
$$\tfrac{1}{2}m\mathrm{v}_f^2 = mg(x_i - x_f)\sin\theta + \tfrac{1}{2}k(x_i^2 - x_f^2)$$

$$\mathrm{v}_f = \sqrt{2g(x_i - x_f)\sin\theta + \frac{k}{m}(x_i^2 - x_f^2)}$$

$$v_f = \sqrt{2(9.8)(0.31-0.14)\sin 30^\circ + \frac{310}{1.7}(0.31^2 - 0.14^2)}$$

$$v_f = \sqrt{1.666 + 13.95} = \boxed{3.95\,\frac{m}{s}}$$

Interesting to plot the potential energies



Conservative and Non-conservative Forces

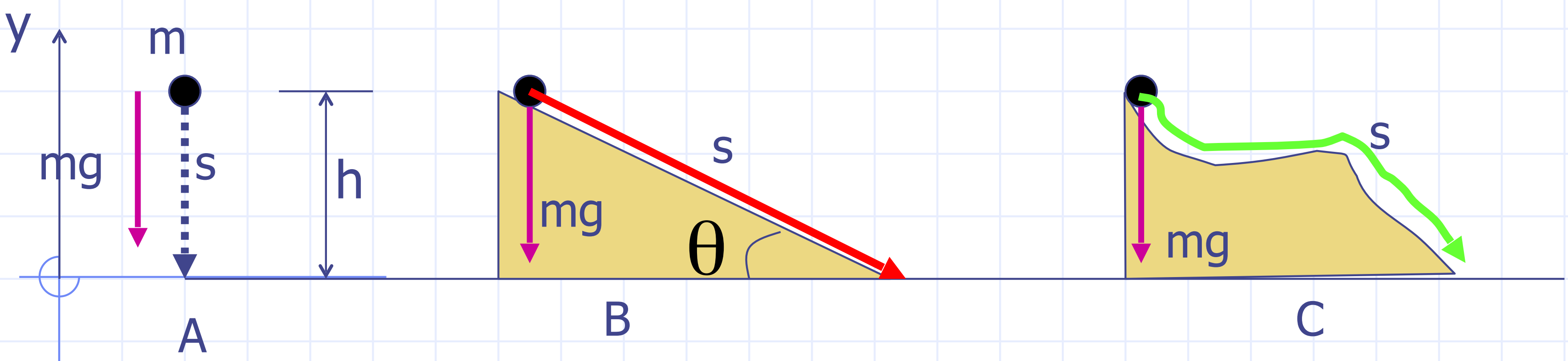
- Conservative Force: a force for which the work it does on an object does not depend on the path. Gravity is an example.

- We know we can obtain the work with the work integral.

$$W = \int_{x_i}^{x_f} F_x dx = W_c$$

- If the force is conservative, then $W = W_c$ and this work can be related to the change in potential energy

$$W_c = \int_{x_i}^{x_f} F_x dx = -\Delta U$$

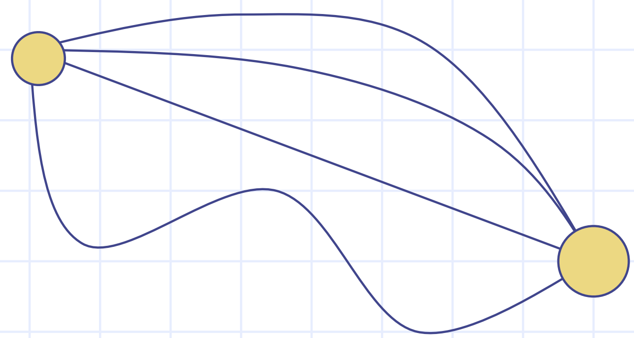


A $$W = F\cos\phi\, s = mgh$$

B $$W = (mg\sin\theta)s = (mg\sin\theta)\frac{h}{\sin\theta} = mgh$$

C $$W = mgh$$

- Non-conservative Force - a force for which the work done depends on the path
- friction
- air resistance



- If the force is conservative, we can find the potential energy due to the force

$$U_f(x) = -\int_{x_i}^{x_f} F_x dx + U_i(x)$$

it is usually convenient to take $U_i(x)=0$

- Or if we know U(x) and the force is conservative, we can obtain F

$$dU(x) = -F_x dx \Rightarrow F_x = -\frac{dU(x)}{dx}$$

- The x-component of a conservative force equals the negative derivative of the potential energy with respect to x

If both conservative and non-conservative forces act on an object, the work-energy theorem is modified

$$W_{total} = W_C + W_{NC} = K_f - K_i$$

$$W_{NC} = K_f - K_i - W_C$$

For the case of gravity

$$W_g = W_C = mg(y_i - y_f)$$

$$W_{NC} = K_f - K_i - mg(y_i - y_f)$$

$$W_{NC} = K_f - K_i - mgy_i + mgy_f$$

$$\boxed{W_{NC} = \Delta K + \Delta U}$$

$$= K_f - K_i + U_f - U_i$$

$$= (K_f + U_f) - (K_i + U_i)$$

$$W_{NC} = E_f - E_i = W_{surr}$$

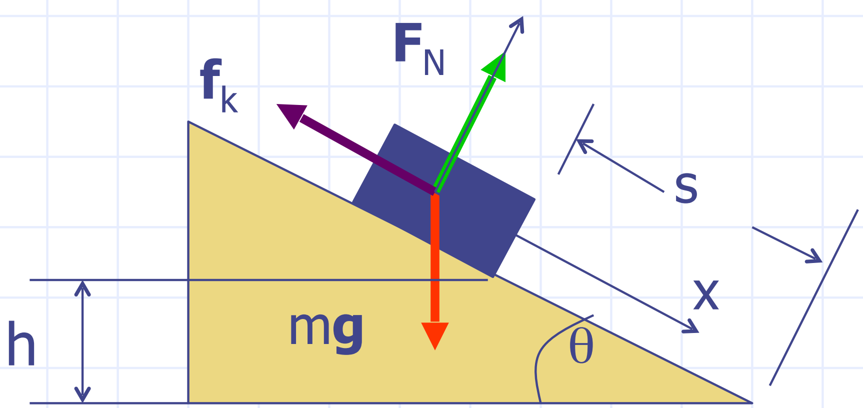
- If no net non-conservative forces

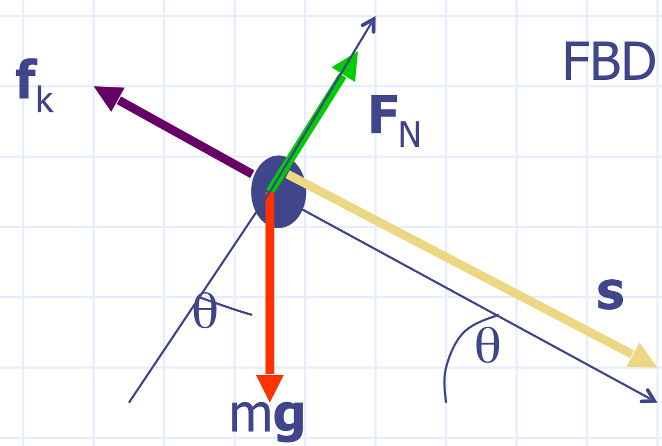
$$W_{NC} = 0 \Rightarrow E_f = E_i = E$$

- Then, conservation of mechanical energy holds

$$\Delta K = -\Delta U$$

Crate on Incline Revisited





$$W_N = 0$$
$$W_g = mg\sin\theta\ s = mgh = W_C$$
$$W_f = -\mu_k mg\cos\theta\ s = W_{NC}$$

- The crate starts from rest, $v_i=0$

$$K_i = 0, E_i = U_i = mgh = W_g$$
$$W_{NC} = E_f - E_i$$
$$E_f = E_i + W_{NC} = mgh - \mu_K mg\cos\theta\ s$$
$$\boxed{E_f < E_i}$$

- Some energy, W_{NC} is loss from the system
- In this case it is due to the non-conservative friction force → energy loss in the form of heat

- Because of friction, the final speed is only 9.3 m/s as we found earlier

- If the incline is frictionless, the final speed would be:

$$E_f = E_i, \text{since } K_i = 0, U_f = 0$$

$$\tfrac{1}{2} m \mathrm{v}_f^2 = mgh = W_g$$

$$\mathrm{v}_f = \sqrt{\frac{2W_g}{m}} = \sqrt{\frac{2(7510\ \mathrm{J})}{100\ \mathrm{kg}}} = \boxed{12.3\ \tfrac{\mathrm{m}}{\mathrm{s}}}$$

- Because of the loss of energy, due to friction, the final velocity is reduced. It seems that energy is not conserved

Conservation of Energy

- There is an overall principle of conservation of energy
- Unlike the principle of conservation of mechanical energy, which can be "broken", this principle **can not**
- It says: ``The total energy of the Universe is, has always been, and always will be constant. Energy can neither be created nor destroyed, only converted from one form to another."
- So far, we have only been concerned with mechanical energy

- There are other forms of energy: heat, electromagnetic, chemical, nuclear, rest mass ($E_m = mc^2$)

$$E_i^{total} = E_f^{total}$$

$$E_i^{mech} + E_i^{others} = E_f^{mech} + E_f^{others}$$

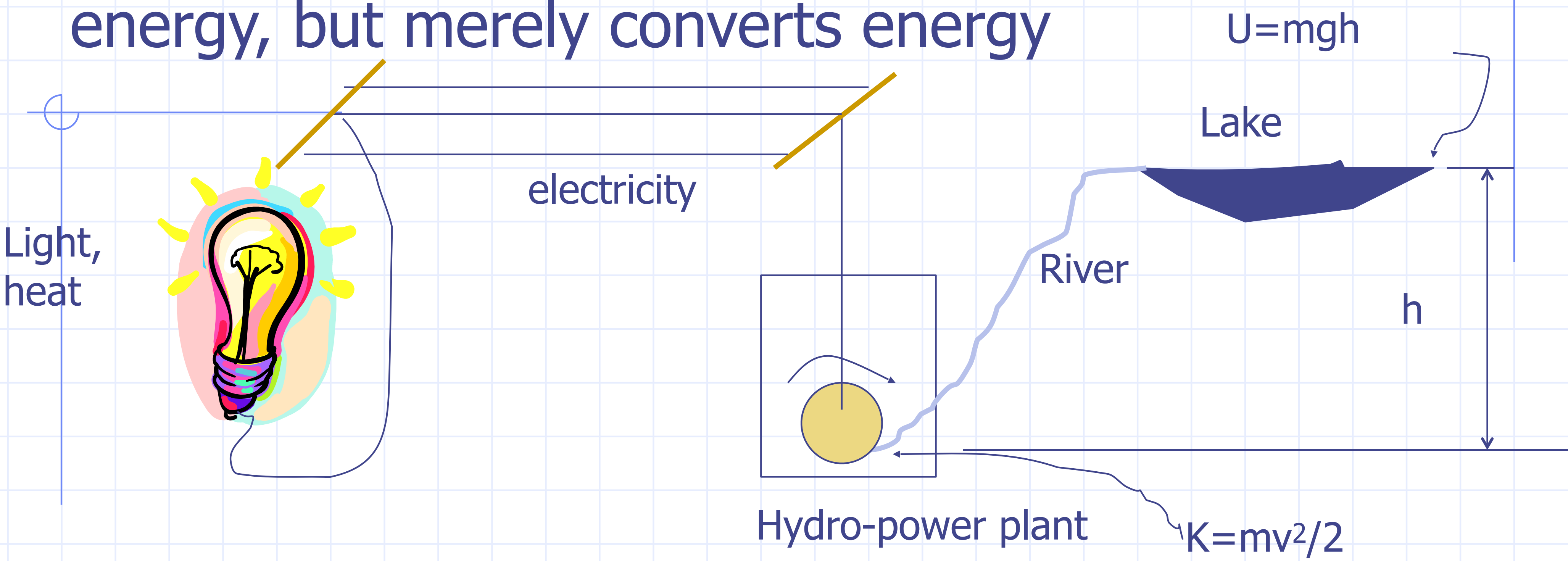
$$E_f^{mech} = E_i^{mech} + E_i^{others} - E_f^{others}$$

$$E_f^{mech} = E_i^{mech} + W_{NC}$$

$$\Rightarrow W_{NC} = E_i^{others} - E_f^{others} = Q$$

- Q (W_{NC}) is the energy lost (or gained) by the mechanical system

- The electrical utility industry does not produce energy, but merely converts energy



Example Problem

A ball is dropped from rest at the top of a 6.10-m tall building, falls straight downward, collides inelastically with the ground, and bounces back. The ball loses 10.0% of its kinetic energy every time it collides with the ground. How many bounces can the ball make and still reach a window sill that is 2.44 m above the ground?

<u>Solution:</u>

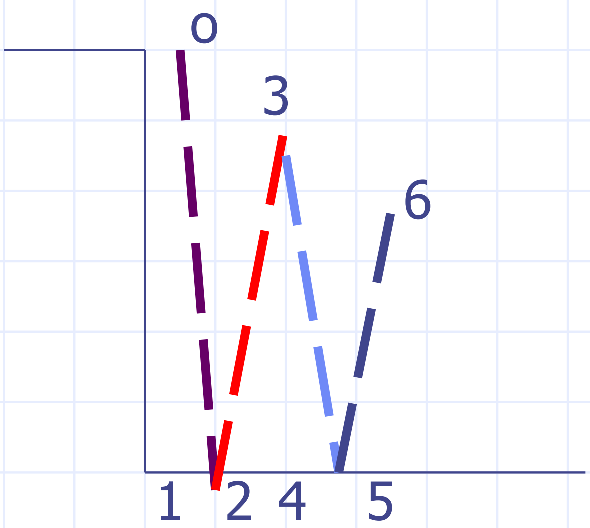
Method: "since the ball bounces on the ground, there is an external force. Therefore, we can **not** use conservation of linear momentum." (Chap. 11)

An inelastic collision means the total energy is not conserved, but we know by how much it is not conserved. On every bounce 10% of K is lost:

$Q_i = 0.1K_i, i = 1,2,3,\ldots,n$ n **is the number of bounces**

Given: h_0 = 6.10 m, h_f = 2.44 m

$$E_o = U_o = mgh_o$$
$$E_1 = K_1 = E_o$$



Since energy is conserved from point 0 to point 1.

However, between point 1 and 2, energy is lost

$$Q_2 = 0.1K_1$$
$$W_{NC} = -Q = E_{final} - E_{initial}$$
$$-Q_2 = E_2 - E_1$$
$$-0.1K_1 = E_2 - E_1$$
$$-0.1E_o = E_2 - E_o$$
$$E_2 = 0.9E_o = E_3 = E_4$$

Total energy after one bounce

By the same reasoning

$$E_5 = 0.9E_4 = 0.9E_2 = 0.9(0.9E_o)$$

$$E_5 = 0.9^2 E_o$$ Total energy after two bounces

The total energy after n bounces is then

$$E_f = 0.9^n E_o$$

$$mgh_f = 0.9^n mgh_o$$

$$h_f = 0.9^n h_o$$

$$0.9^n = h_f / h_o$$

$$\log(0.9)^n = \log(h_f / h_o)$$

$$n \log(0.9) = \log(h_f / h_o)$$

$$n = \frac{\log(h_f / h_o)}{\log(0.9)}$$

$$n = \frac{\log(2.44 / 6.10)}{\log(0.9)}$$

$$n = 8.7$$

Answer is 8 bounces