Chapter 10

Potential Energy and Conservation of Energy

Work Done by Gravity

- If one lifts an object of mass m from the floor (y_i=0) to a height y_f=h, you have done work on the object $$W = F\cos\phi\, s = mg(y_f - y_i) = mgh$$
- We have imparted energy to it, but it is at rest (v=0). So, this energy is not kinetic energy. It is called Potential Energy (PE or U), or in this particular case, gravitational potential energy
- U is energy that is stored and which can be converted to another kind of energy, K for example

$$U_g = mgy$$

- U is a scalar with units of J in S.I.
- y is the height above some reference point, e.g. table, floor, …

Conservation of (Mechanical) Energy

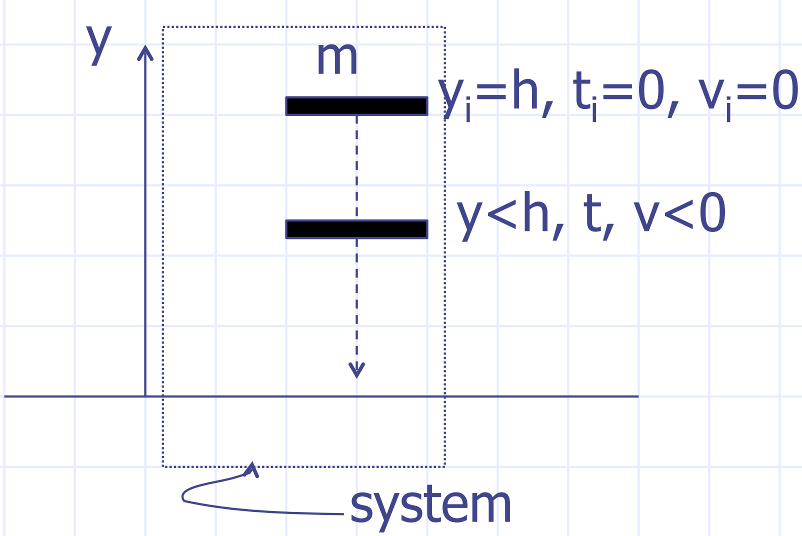
- Total (mechanical) energy is constant within some specified system
  - total energy is conserved
  - conservation principles are very important in physics; we will see many others later

$$E = K + U$$

- E is always constant, but K and U can change
- If U and K change, they must change in such a way as to keep E constant

Example

Consider the 1D free-fall of an object of mass m from a height of $y_i=h$



The initial energy of the system defines the total energy

$$U_i = mgh, \quad K_i = 0$$

$$E = K + U = mgh$$

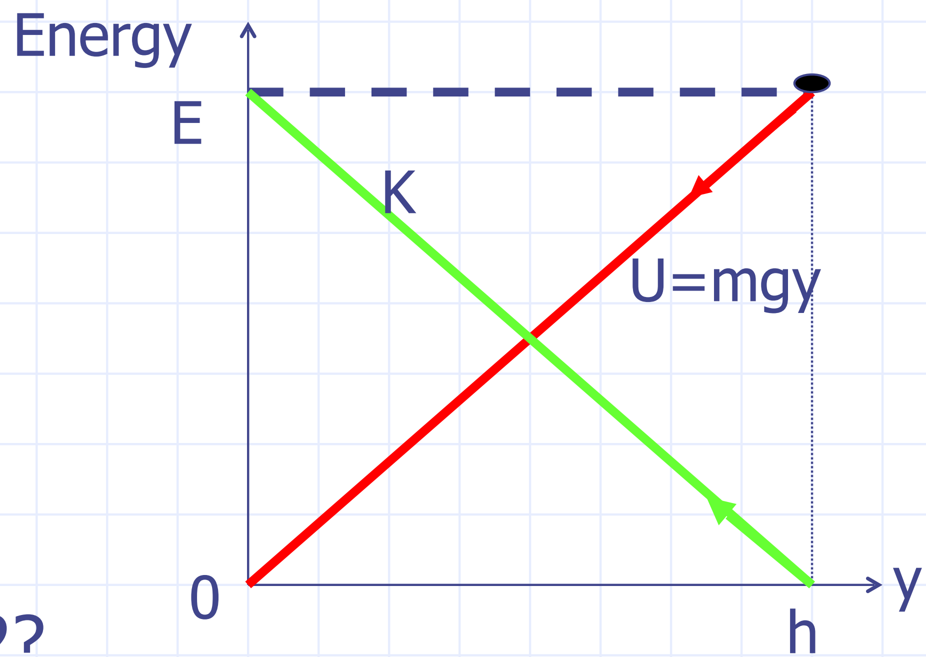
- System – the collection of objects being study to the exclusion of all other objects in the surroundings, in this example, we consider the object of mass m only
- Some time later, K and U have changed, but E has not

$$E = K + U$$

$$mgh = \tfrac{1}{2} m\mathrm{v}^2 + mgy$$

$$K = mgh - mgy$$

$$= mg(h - y)$$



What is K, U, and v when y=h/2?

$$U = mgy = \boxed{mg\frac{h}{2}}, \quad K = mg(h - \frac{h}{2}) = \boxed{mg\frac{h}{2}}$$

$$K = mg\frac{h}{2} = \tfrac{1}{2}mv^2$$

$$v = \sqrt{gh} = \sqrt{(9.80\,\tfrac{m}{s^2})(1.00\text{ m})} = \boxed{3.13\,\tfrac{m}{s}}$$

$$E = mgh = (1.00\text{ kg})(9.80\,\tfrac{m}{s^2})(1.00\text{ m}) = \boxed{9.80\text{ J}}$$

$$U = K = mg\frac{h}{2} = \frac{E}{2} = \boxed{4.90\text{ J}}$$

For y=0 (just before the object hits the ground)?

$$U = mgy = \boxed{0},\ \ E = K = mgh = \boxed{9.80\text{ J}}$$

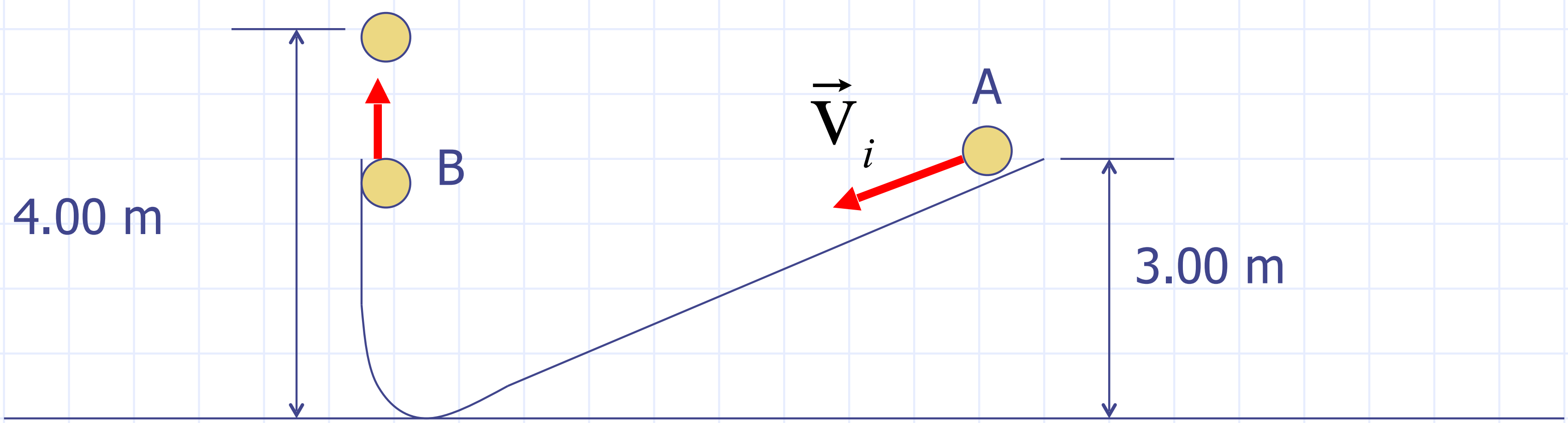
$$\tfrac{1}{2}mv^2 = mgh$$

$$v = \sqrt{2gh} = \sqrt{2(9.80\,\tfrac{m}{s^2})(1.00\text{ m})} = \boxed{4.43\,\tfrac{m}{s}}$$

Note: we have neglected air resistance and what happens when the object hits the ground.

Example Problem

A particle starting from point A, is projected down the curved runway. Upon leaving the runway at point B, the particle is traveling straight upward and reaches a height of 4.00 m above the floor before falling back down. Ignoring friction and air resistance, find the speed of the particle at point A.



Example Problem

A grappling hook, attached to a 1.5-m rope, is whirled in a circle that lies in the vertical plane. The lowest point on this circle is at ground level. The hook is whirled at a constant rate of three revolutions per second. In the absence of air resistance, to what maximum height can the hook be cast?

Method: use concepts of conservation of mechanical energy and uniform circular motion