Chapter 9: Energy and Work

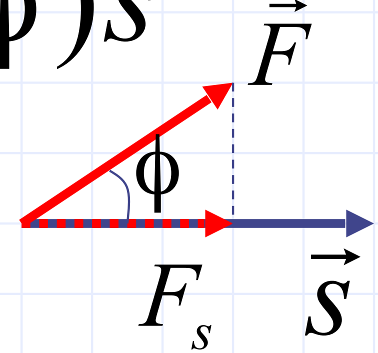
- Alternative method for the study of motion
- In many ways easier, gives additional information
- Kinetic energy: consider an object of mass m and speed v, we define the kinetic energy as

$$E_K = \frac{1}{2} m \mathrm{v}^2 = K$$

- a scalar, not a vector
- units kg m^2/s^2 = N m = Joule (J) in S.I. (ft lb in B.E. and erg in CGS)
- like speed, gives a measure of an object's motion (a car and tractor-trailer may have the same v, but different K)

Work: the work done on an object by an applied constant net force **F** which results in the object undergoing a displacement of **s** (or **x** or **r**)

$$W = \vec{F}\cdot\vec{s} = F_s s = (F\cos\phi)s$$
$$= F(s\cos\phi)$$



- a scalar, units of N m = J
- if **F** and **s** are perpendicular, W=0
- work can be negative (ϕ>90°)

Work-Energy Theorem: when a net external force does work on an object, there is a change in the object's KE

$$\sum W = W_{total} = \Delta K = K_f - K_i$$

$$W_{total} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

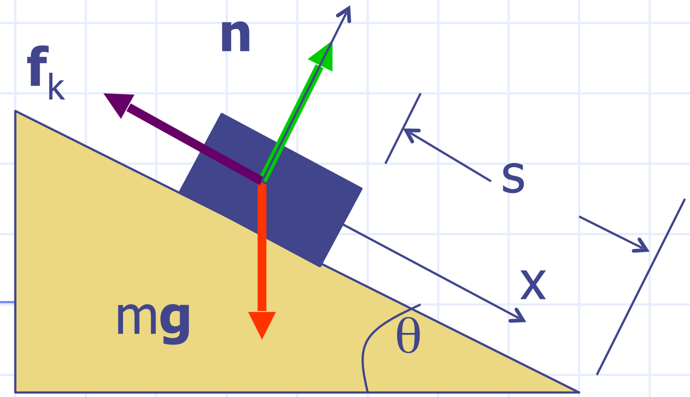
Example

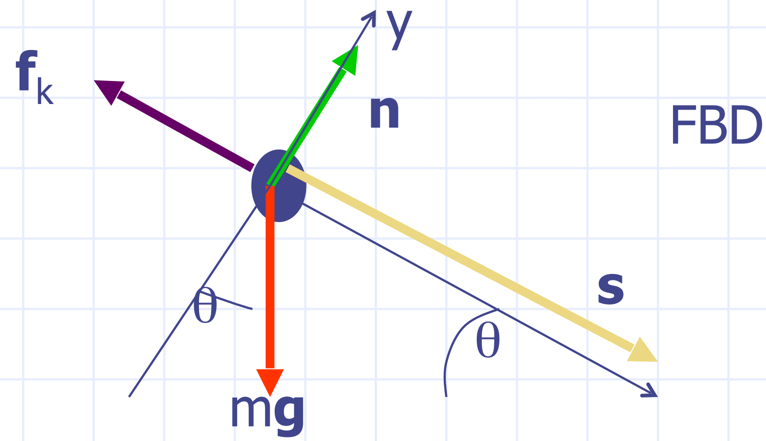
A crate on a incline is held in place by a rope. The rope is released and the crate slides to the bottom. Determine the total work done if the crate has a mass of 100 kg, the incline has angle of 50.0°, the coefficient of kinetic friction is 0.500, and displacement of the crate is 10.0 m.

Solution:

Given: m = 100 kg, θ = 50°, μ_k = 0.500, s = 10.0 m

Approach: compute the work for each force





- Only force components along the direction of **s** contribute (x-direction)

$$\sum F_y = n - mg\cos\theta = 0$$

$$n = mg\cos\theta$$

$$W_N = n\cos 90^\circ s = 0$$

$$W_f = F\cos\phi\,\mathrm{s} = f_k\cos 180^\circ s = -f_k s$$

$$= -\mu_k mg\cos\theta\,\mathrm{s} = \boxed{-3.15\mathrm{x}10^3\ \mathrm{J}}$$

$$W_g = mg\cos(90^\circ - \theta)s = mg\sin\theta\ s$$

$$= 7.51\text{x}10^3\ \text{J}$$

$$\text{Total work} = W_{total} = W_g + W_f$$

$$= mg\sin\theta\ s - \mu_k mg\cos\theta\ s$$

$$= mgs(\sin\theta - \mu_k\cos\theta)$$

$$= 7.51\text{x}10^3 - 3.15\text{x}10^3\ \text{J} = 4.36\text{x}10^3\ \text{J}$$

- Or calculate the net force along **s** (x-direction)

$$\sum F_x = mg\sin\theta - f_k = mg\sin\theta - \mu_k mg\cos\theta$$

$$= mg(\sin\theta - \mu_k\cos\theta) = F_s\ \ (= ma_x)$$

$$W = F_s s = mgs(\sin\theta - \mu_k\cos\theta)$$ Same as above

- Now determine final velocity from work-energy theorem, since $v_i = 0$, $K_i = 0$

$$W_{total} = K_f - K_i = \tfrac{1}{2} m v_f^2$$

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(4360\ \text{J})}{100\ \text{kg}}} = \boxed{9.34\ \tfrac{\text{m}}{\text{s}}}$$

- Check by kinematics

$$v_f^2 = v_i^2 + 2a_x x = v_i^2 + 2a_s s$$

$$v_f = \sqrt{2a_s s} = \sqrt{2g(\sin\theta - \mu_k \cos\theta)s}$$

$$v_f = \boxed{9.34\ \tfrac{\text{m}}{\text{s}}}$$

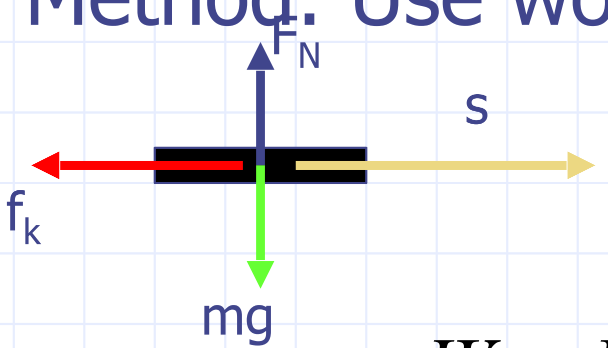
Example

A hockey puck slides across the ice. Its speed slows from 45.00 m/s to 44.67 m/s after traveling a distance of 16.0 m. Determine the coefficient of kinetic friction between the ice and the puck.

Solution:

Given: v_i=45.00 m/s, v_f=44.67 m/s, x=16.0 m=s

Method: Use work-energy theorem



$$\sum F_y = F_N - mg = 0 \Rightarrow F_N = mg$$

$$\sum F_x = -f_k = ma_x, \; f_k = \mu_k F_N = \mu_k mg$$

$$W = F \cos\phi \; s = f_k \cos 180^\circ s = -f_k s = -\mu_k mgs$$

$$W = K_f - K_i$$

$$= \tfrac{1}{2}m\mathrm{v}_f^2 - \tfrac{1}{2}m\mathrm{v}_\mathrm{i}^2 = -\mu_k mgs$$

$$\mathrm{v}_f^2 - \mathrm{v}_\mathrm{i}^2 = -2\mu_k gs$$

$$\mu_k = -\frac{(\mathrm{v}_f^2 - \mathrm{v}_\mathrm{i}^2)}{2gs} = -\frac{(44.67^2 - 45.00^2)}{2(9.80)(16.0)}$$

$$= \boxed{0.094}$$